BESOV SEMI-NORM PENALTY METHOD FOR SIGNAL RESTORATION

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ABSTRACT. In this paper, we propose an adaptive iteration algorithm for signal/image restoration. For digital signals, this algorithm minimizes a piecewise nonlinear l_q, $0 < q \leq 1$, functional defined on wavelet domain with a single equality constraint. This minimization algorithm is based on the Besov semi-norm penalty method. Rudin et al's BV semi-norm based method is practically a piecewise linear l_1 minimization algorithm on the spatial difference domain, so that it has a great success for reconstruction of piecewise constant signals. Our proposed method provides a non-oscillatory and edge preserving algorithm for reconstruction of piecewise smooth signals.

1. INTRODUCTION

In this paper, we propose an adaptive iteration algorithm for signal/image restoration. For digital signals, this algorithm minimizes a piecewise nonlinear l_q, $0 < q \leq 1$, functional defined on the wavelet domain with a single equality constraint. This minimization algorithm is from a Besov semi-norm penalty method and corresponds to a restoration process. In practice, restoration and enhancement processes have many applications in information technology areas such as digital communications, CCD/CCTV imaging, medical imaging, and astronomy telescope imaging.

Observed or acquired data through a mechanical/communication system are usually blurred and noisy. The restoration and enhancement of blurred and noisy data are ill-posed inverse problems in theoretical and computational point of views. Mathematically, an acquired signal is modeled via an integral equation:

$$g = Af + w \tag{1}$$

where g is the acquired signal which is degraded by a blurring operator A and a random noise w from original signal f. In signal/image processing applications, A is usually modeled as a convolution operator and w a Gaussian white random process(cf. [2], [15]). For instance, A is represented by the Gaussian kernel function $k(x, y) = \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(x-y)}{\sigma^2}}$ as

$$Af(x) = \int k(x, y) f(y)\, dy.$$

In this case, A is a compact operator or generally a Fredholm first kind integral operator, so that the problems of the form (1) are ill-posed problems in the sense of Hadamard. In both theory and computations, it is very difficult solve ill-posed problems.

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Many image restoration methods have been recently proposed(cf. [11]). One basic approach is to formulate the restoration problem as a constrained least squares problem. This approach is a regularization method. A general form of regularization for the problem (1) is of the constrained minimization problem with respect to f:

$$\begin{aligned} &\min \quad J(f) \\ &\text{subject to} \quad \|Af - g\|_{L_2} = \sigma \end{aligned} \tag{2}$$

where J is a certain objective functional, σ is the standard deviation of the noise w, and $\|\cdot\|_{L_2}$ denotes the L_2-norm. There are several possible choices for the objective functional J, for instance, the L_2-norm of the gradient ∇f or some higher derivatives of f. For signal/image processing applications, Rudin et al. [14], [15] have recently suggested the use of the bounded variation (BV) semi-norm $J(f) = |f|_{BV}$. With this choice, a solution to (2) provides an image with the least total variation among all the images with the standard deviation σ, so that the bounded variation based restoration algorithm has a great success in the recovery of corrupted piecewise constant images such as blocky images.

Rudin et al.'s BV based method has, however, a disadvantage for the piecewise smooth signals/images. In this paper, a numerical simulation is presented to demonstrate the BV based method fails(see §4). In addition, we propose a Besov semi-norm(cf. [17]) regularization method for the problem (2) to overcome the disadvantage while keeping non-oscillatory(minimal ringing) and non-invasive(edge preserving) reconstruction.

The minimization problem (2) with $J(f) = |f|_{BV}$ can be solved by finding the steady state of the nonlinear diffusion process driven by the Euler-Lagrange equation(cf. [14], [15]). Because of the complicate nonlinear structure of Besov semi-norm(see §2), the minimization problem (2) with Besov semi-norm

$$\begin{aligned} &\min \quad |f|_{B_q^\alpha} \\ &\text{subject to} \quad \|Af - g\|_{L_2} = \sigma \end{aligned} \tag{3}$$

can not be handled by the Euler-Lagrange formulation. However, in computation, the minimization problem (3) can be considered as an l_q-norm minimization problem. In [9], [7], the Besov semi-norm is characterized in terms of a weighted l_q-norm of wavelet coefficients. Using the wavelet coefficient characterization of the signal f, we reformulate the minimization problem (3) as a weighted l_q-norm problem with a single equality constraint(see §3).

For $0 < q \leq 1$, the l_q-norm problem is a nonlinear piecewise differentiable minimization problem with singularity at the origin. So, it is difficult to reach an optimal solution to the minimization problem. There has recently been reports [12], [4] for a globally convergent method for l_q-norm problems. In [13], the authors have proposed a reliable and efficient computational algorithm based on the BV semi-norm reguarization method. This algorithm basically minimizes a piecewise linear l_1 objective functional(a measure of bounded variation). For solving this constrained l_1 minimization problem, an affine scaling Newton method is employed in [12], [13]. We adapt the affine scaling Newton method for solving our l_q-norm minimization problem(see §4).

For our numerical simulation, we generate blurred and noisy signal data by convolving a known signal with a given blurring function, and adding measured amounts of random noise(see §4).

2. Besov semi-norm regularization

The Tikhonov regularization [16] is a common and classical method to find a reasonable solution to ill-posed inverse problem. For signal/image processing applications, however, this regularization has drawbacks such that it produces a large oscillatory solution. Regularization with semi-norm is a reasonable approach for signal/image restoration(cf. [3]). The BV semi-norm regularization has a great success for restoration of piecewise constant signals/images(cf. [13], [1]). In this section, we consider a regularization with Besov semi-norm for the problem (1).

The Besov space can be defined via the modulus of smoothness(cf. [17]). Let Ω be a domain in d-dimensional Euclidean space $\mathbb{R}^d$. The modulus of smoothness of order r of $f \in L_p(\Omega)$, $0 < p \le \infty$, is defined by

$$\omega_r(f,t)_p := \sup_{|h|<t} \|\Delta_h^r(f,\cdot)\|_{L_p}, \quad t > 0$$

where $\Delta_h^r(f,x)$ is the kth difference of f in the direction $h \in \mathbb{R}^d$.

Definition 2.1. *Let $0 < \alpha < \infty$, $0 < p, q \le \infty$, and r be a positive integer with $\alpha < r$. The Besov space $B^\alpha_{p,q}(\Omega)$ is is defined as the set of all functions $f \in L_p(\Omega)$ for which*

$$|f|_{B^\alpha_{p,q}} := \Big(\sum_{\nu\ge 0}\big[2^{\nu\alpha}\omega_r(f,t)_p\big]^q\Big)^{1/q}$$

is finite. A (quasi-) norm for $B^\alpha_{p,q}(\Omega)$ is defined by

$$\|f\|_{B^\alpha_{p,q}} := \|f\|_{L_p} + |f|_{B^\alpha_{p,q}}.$$

Remark. It is known [17] that all values of $r > \alpha$ result in equivalent (quasi-) norms of Besov space. In the definition above, $|\cdot|_{B^\alpha_{p,q}}$ is a semi-norm which annihilates all polynomial g with degree $\lceil\alpha\rceil$, i.e.,

$$|g|_{B^\alpha_{p,q}} = 0.$$

For the case $1 < \alpha < 2$, we shall employ an equivalent semi-norm

$$|f|_{B^\alpha_{p,q}} \approx |f|_{W^1_p} + \sum_{j=1}^d \Big|\frac{\partial f}{\partial x_j}\Big|_{B^\alpha_{p,q}} \tag{4}$$

where W^1_p is a Sobolev space(cf. [17]).

The family $B^\alpha_q := B^\alpha_{q,q}(\Omega)$, $\frac{1}{q} = \frac{\alpha}{d} + \frac{1}{p}$, of Besov spaces has practical applications such as to nonlinear approximation theory and image compression(cf. [6]). The report [6] has argued that images with edges have more smoothness in the Besov space than in other function spaces such as the Sobolev space. With this motivation, we propose the Besov semi-norm minimization problem (3). In this paper, we focus on an efficient computation algorithm for the problem (3).

We regard a signal as a function on $\Omega = [0,1]$. A function with a jump discontinuity(or edge) can be a member of the space B^α_q with certain range of the smoothness parameter α(cf. [6]). For signal restoration, we confine the parameters in the space B^α_q as

$$p = 2, \quad 0 < \alpha < 2, \quad d = 1,$$

so that $q = \frac{2}{2\alpha+1}$. If the parameter α is greater than 1/2, then $0 < q < 1$. ¿From the embedding property of Besov spaces [8], the space B_q^α is continuously embedded in L_2, that is, $B_q^\alpha \subset L_q \cap L_2$.

Since it is difficult to drive an Euler-Lagrange equation for the minimization problem, we employ the wavelet decomposition of the space B_q^α. Let ψ be a wavelet for L_2 with decomposition

$$f = \sum_k \sum_{j\in\Lambda_k} c_{k,j}\psi_{k,j} \tag{5}$$

where $\psi_{k,j}(x) := 2^{k/2}\psi(2^k x - j)$, $k \in \mathbb{N}$, $j \in \Lambda_k$. Here Λ_k is the index set for the wavelet coefficients at the resolution level k. It is known [5] that an equivalent norm to the L_2 norm of f is given by the wavelet coefficients, that is, $\|f\|_{L_2}^2 \approx \sum_{k,j} |c_{k,j}|^2$. Furthermore, the Besov norm of f is characterized in terms of the coefficients as follows [9], [8]:

$$\|f\|_{B_q^\alpha}^q \approx \sum_{k,j} |c_{k,j}|^q \tag{6}$$

where $q = \frac{2}{2\alpha+1}$. In addition, if $d_{k,j}$ are wavelet coefficients of ∇f, that is,

$$\nabla f = \sum_{k,j} d_{k,j}\psi_{k,j}, \tag{7}$$

then

$$\|\nabla f\|_{B_q^{\alpha-1}}^q \approx \sum_{k,j} (2^{-k}|d_{k,j}|)^q. \tag{8}$$

If the decomposition (7) is stopped at a low resolution level $k = k_0 > 0$, then

$$\nabla f = \sum_{j\in\Lambda_{k_0}} c_{k_0,j}\phi_{k_0,j} + \sum_{k\geq k_0}\sum_{j\in\Lambda_k} d_{k,j}\psi_{k,j} \tag{9}$$

and the equivalence (8) has the form

$$||\nabla f||_{B_q^{\alpha-1}}^q \approx \sum_j (2^{-k\alpha}|c_{k_0,j}|)^q + \sum_{k\geq k_0}\sum_j (2^{-k}|d_{k,j}|)^q, \tag{10}$$

where ϕ is the scaling function associated with the wavelet ψ. We adopt the right hand side of (10) as the semi-norm $\|\nabla f\|_{B_q^{\alpha-1}}^q$. Thus we obtain an equivalent minimization problem to (3) as follows:

$$\begin{aligned} &\min \quad \|\nabla f\|_{B_q^{\alpha-1}}^q \\ &\text{subject to} \quad \|Af - g\|_{L_2}^2 = \sigma^2. \end{aligned} \tag{11}$$

3. Wavelet discretization

In computation, a digital signal is given, so that we then consider the discretization of (11). First of all, the discretizations of A, f, and g are as follows:

- K**:** a finest resolution level so that $2^K \simeq N$.
- $\boldsymbol{u}$, $\boldsymbol{v} \in \mathbb{R}^N$**:** discrete representations of f and g, respectively.
- $\boldsymbol{A} \in \mathbb{R}^{N\times N}$**:** a matrix representation of A.

To discretize $\|\nabla f\|_{B_q^{\alpha-1}}$ with the wavelet decomposition, let us recall from [5] the refinement equation for the scaling function ϕ,

$$\phi(x) := \sum_j a_j \sqrt{2}\, \phi(2x-j), \tag{12}$$

and the Mallat formula for the wavelet ψ,

$$\psi(x) := \sum_j b_j \sqrt{2}\, \phi(2x-j), \quad b_j = (-1)^j a_{1-j} \tag{13}$$

where a_j is the scaling coefficients of ϕ. The discrete wavelet transform is given by the pair

$$\begin{aligned} H &:= (h_{j,l}), \quad h_{j,l} := a_{l-2j} \\ G &:= (g_{j,l}), \quad g_{j,l} := b_{l-2j}. \end{aligned}$$

Let $\boldsymbol{c}_k = \{c_{k,j}\}$ be the approximation coefficients of $\boldsymbol{u}$ with respect to $\phi_{k,j}$ and $\boldsymbol{d}_k = \{d_{k,j}\}$ wavelet coefficients of $\boldsymbol{u}$ with respect to $\psi_{k,j}$ at a level k. Then

$$\boldsymbol{c}_k = H\boldsymbol{c}_{k+1}, \quad \boldsymbol{d}_k = G\boldsymbol{c}_{k+1}.$$

Moreover, a given approximation coefficients $\boldsymbol{c}_K$ at the level K,

$$\boldsymbol{c}_{k_0} = H^{K-k_0}\boldsymbol{c}_K, \quad \boldsymbol{d}_{k_0} = GH^{K-k_0-1}\boldsymbol{c}_K, \qquad k_0 < K. \tag{14}$$

If we let $\boldsymbol{D}$ be a matrix representation of ∇, then the wavelet coefficient vector for $\boldsymbol{Du}$ is given by

$$\begin{bmatrix} \boldsymbol{c}_{k_0} \\ \boldsymbol{d}_{k_0} \\ \vdots \\ \boldsymbol{d}_{K-1} \end{bmatrix} = \boldsymbol{WDu}, \qquad \text{where} \quad \boldsymbol{W} := \begin{bmatrix} H^{K-k_0} \\ GH^{K-k_0-1} \\ \vdots \\ GH \\ G \end{bmatrix}.$$

In addition, let

$$\boldsymbol{B} := \operatorname{diag}\left[(2^{-k_0\alpha}\boldsymbol{e}_{k_0}, 2^{-k_0\alpha}\boldsymbol{e}_{k_0}, \cdots, 2^{-(K-1)\alpha}\boldsymbol{e}_{K-1})\right]\boldsymbol{WD}, \tag{15}$$

where $\boldsymbol{e}_k \in \mathbb{R}^{\#(\Lambda_k)}$ is the one vector $(1, 1, \cdots, 1)$. Therefore the discrete representation of $\|\nabla f\|^q_{B^{\alpha-1}_{q,q}}$ is $\|\boldsymbol{Bu}\|_q^q$, where $\|\cdot\|_q$ is the l_q (quasi-)norm. Thus, a wavelet discretization of (11) is given by

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & \|\boldsymbol{Bu}\|_q^q \\ \text{subject to} \quad & \|\boldsymbol{Au} - \boldsymbol{v}\|_2^2 = \sigma^2. \end{aligned} \tag{16}$$

4. l_q MINIMIZATION PROBLEMS

The problem (16) is an l_q minimization problem with a single quadratic equality constraint:

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & J(\boldsymbol{u}) := \|\boldsymbol{Bu}\|_q^q \\ \text{subject to} \quad & \|\boldsymbol{Au} - \boldsymbol{v}\|_2^2 = \sigma^2. \end{aligned} \tag{17}$$

The l_q minimization problem has been studied by many researchers. For $q = 2$, the replacement $\boldsymbol{Bu}$ with $\boldsymbol{Bu} - \boldsymbol{c}$ is a classical least squares problem. If $1 \le q \le \infty$ and $\boldsymbol{B}$ is a Toepiltz matrix, then it has an application to image processing and is extensively studied by Coleman et al. [4], [12]. For the matrix $\boldsymbol{B}$ of ∇, and $q = 1$, it is the BV minimization problem(cf. [13]). In our case, $\boldsymbol{B}$ is given by (15) and $0 < q \le 1$.

Outline of a computational algorithm for solving the l_q problem is as follows:

- **Step1 :** Choose an initial signal $\boldsymbol{u}_0$ within the feasibility.
- **Step2 :** Determine a new signal $\boldsymbol{u}_{n+1}$ with the form $\boldsymbol{u}_{n+1} = \boldsymbol{u}_n + \delta_n \boldsymbol{s}_n$ so that
$$J(\boldsymbol{u}_{n+1}) < J(\boldsymbol{u}_n), \qquad \|\boldsymbol{A}\boldsymbol{u}_{n+1} - \boldsymbol{v}\|_2^2 = \sigma^2,$$
where $\boldsymbol{s}_n$ is a suitable descent direction and δ_n a suitable step length.
- **Step3 :** Given a tolerance τ, stop for
$$\frac{J(\boldsymbol{u}_n) - J(\boldsymbol{u}_{n+1})}{|J(\boldsymbol{u}_{n+1})|} \le \tau.$$

There are some difficulties in solving the problem (17) for $0 < q \le 1$ in the practical point of view. First, due to the non-differentiability of the objective functional $J(\boldsymbol{u})$, the algorithm can fail to converge or converge extremely slowly. Secondly, with a naive choice of the descent direction $\boldsymbol{s}_n$ and the step length δ_n, it is difficult to maintain the feasibility.

In **Step 1**, by applying a CG(conjugate gradient) method, a good initial signal $\boldsymbol{u}_0$ can be obtained so that $\|\boldsymbol{A}\boldsymbol{u}_{\boldsymbol{0}} - \boldsymbol{v}\|_2^2 = \sigma^2$. In **Step 2**, the descent direction $\boldsymbol{s}_n$ needs to satisfy $\nabla J(\boldsymbol{u}_n)^T \boldsymbol{s}_n < 0$. For a robust descent direction, Coleman et al. [4] have suggested the affine scaling method that is an efficient interior point method, especially for the l_q problems. Once the direction $\boldsymbol{s}_n$ is determined, the step length δ_n can be computed as the minimizer of $\min_{\delta > 0} J(\boldsymbol{u}_n + \delta \boldsymbol{s}_n)$. In this case, a suitable line search procedure needs to be considered for reducing the a piecewise concave functional J with break points and maintaining the feasibility(cf. [13]).

In order to obtain a good descent direction $\boldsymbol{s}_n$ for our algorithm, we adapt the affine scaling Newton method of Coleman et al. [4] with incorporating the constraint $\|\boldsymbol{A}\boldsymbol{u} - \boldsymbol{v}\|_2^2 = \sigma^2$(cf. [13]).

Let $\boldsymbol{r}$ be the residual such that

$$\boldsymbol{r} := \begin{bmatrix} \|\boldsymbol{A}\boldsymbol{u} - \boldsymbol{v}\|_2^2 - \sigma^2 \\ \boldsymbol{B}\boldsymbol{u} \end{bmatrix}, \quad \boldsymbol{g} := \begin{bmatrix} 0 \\ q|\boldsymbol{B}\boldsymbol{u}|^{q-1}\mathrm{sgn}\,(\boldsymbol{B}\boldsymbol{u}) \end{bmatrix} \tag{18}$$

where $\mathrm{sgn}\,(\boldsymbol{c}) := (\mathrm{sgn}\,(c_j))$ and $|\boldsymbol{c}|^p := (|c_j|^p)$. The dual problem capturing optimality of (16) is the nonlinear system

$$\begin{aligned} \boldsymbol{L}\boldsymbol{\lambda} &= \boldsymbol{0}, \\ \mathrm{diag}[\boldsymbol{r}](\boldsymbol{g} - \boldsymbol{\lambda}) &= \boldsymbol{0}, \end{aligned} \tag{19}$$

where $\boldsymbol{L} := [\boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{u} - \boldsymbol{v}), \boldsymbol{B}^T]$(cf. [4]). Here $\boldsymbol{\lambda}$ is the dual multiplier for (16). Form the Newton method for (19), we obtain

$$\begin{bmatrix} \lambda_{n_1}\boldsymbol{A}^T\boldsymbol{A} & \boldsymbol{L}_n \\ \mathrm{diag}[q\boldsymbol{g}_n - \boldsymbol{\lambda}_n]\boldsymbol{L}_n^T & -\mathrm{diag}[\boldsymbol{r}_n] \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_n \\ \boldsymbol{\lambda}_{n+1} \end{bmatrix} = - \begin{bmatrix} \boldsymbol{0} \\ \mathrm{diag}[\boldsymbol{r}_n]\boldsymbol{g}_n \end{bmatrix}. \tag{20}$$

At each iteration, the matrix of (20) can be nearly singular, so that we adapt Coleman's globalization [4] of the Newton step.

For an instant numerical simulation, let f be defined on $\Omega = [0, 1]$ represented by the vector $\boldsymbol{u} \in \mathbb{R}^N$ with $N = 2^K + 1$, $K = 11$(see Figure 1(a)). Let $\boldsymbol{A}$ be the matrix representation of A with the Gaussian kernel $k(x,y)$:

$$Af(x) = \int_{\Omega} k(x,y)f(y)dy, \quad k(x,y) = \frac{1}{\sqrt{\pi}\omega} e^{-\frac{x-y}{\omega^2}}$$

where $\omega = 0.05$. Also, let $\boldsymbol{w}$ be the vector representation of the additive noise w such that

$$\boldsymbol{w} \overset{\text{iid}}{\backsim} N(0,\sigma), \quad \text{where} \quad \frac{\sigma\sqrt{N}}{\|\boldsymbol{Au}\|_2} = 0.05.$$

In Figure 1, two restorations of a blurred and noisy signal are presented. Figure 1(c) shows a better reconstruction of the Besov semi-norm method near both linear and constant areas than that of the BV method(see Figure 1(d)). For more detailed numerical results, we refer to [10].

FIGURE 1. The dotted lines in (a), (c), and (d) show the exact signal. (a) Blurred signal, (b) Blurred and noisy signal, (c) The reconstructed signal by the Besov semi-norm based method with $\alpha = 3/2$, (d) The reconstructed signal by the BV based method.

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