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ON THE IMPLIED VOLATILITIES AND RISK PREMIUMS

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Triggered by the seminal work of Black and Scholes (1973), the financial engineering has been rapidly developed not only in terms of the market size, but also in terms of the variety of products. The rapid expansion of financial engineering led to development of theoretical and empirical researches on derivatives securities, producing a tremendous amount of literature. Among them, one important issue is to exactly measure and to efficiently forecast the future volatilities of underlying assets. As is widely known, the volatility parameter is not directly observable in the market, and thus it should be estimated to evaluate option prices or to hedge risky positions with options. Empirically, there are two possible alternatives to estimate it. The first is the "implied volatility" that is reverse-engineered using option prices equal to the model prices, and the second is the "historical volatility" that is estimated from the past time-series of underlying asset returns.

Usually, studies that compare the informational efficiency of two alternatives adopt the following univariate and encompassing regressions.

$$\begin{aligned}
\sigma_R(t) &= \alpha + \beta\sigma_H(t) + \epsilon \\
\sigma_R(t) &= \alpha + \beta\sigma_I(t) + \epsilon \\
\sigma_R(t) &= \alpha + \beta\sigma_I(t) + \gamma\sigma_H(t) + \epsilon(t)
\end{aligned}$$

where $\sigma_R(t)$ is the annualized realized volatility of the underlying asset at time t during the period between time t and the expiration date, $\sigma_I(t)$ is the annualized implied volatility at time t, $\sigma_H(t)$ is the annualized ex-post historical volatility of the underlying asset at time t during the period between past certain point and time t, and $\epsilon(t)$ is the forecasting error. If the value of β is statistically significant and positive, it means that the volatility estimator is informative pertinent to future volatility. Also, if a volatility forecast is the conditional expectation of market participants for future volatilities, it will be an unbiased estimator of future volatilities and thus $\{\ \alpha, \beta\ \}$ should converge to $0, 1$. Finally, if one volatility forecaster contains superior information regarding future volatilities, it will be an unbiased estimator even if the other is added. In other words, $\{\alpha = 0, \beta = 1, \gamma = 0\}$ or $\{\alpha = 0, \beta = 0, \gamma = 1\}$ has to be held.

Subsidiarily, they also adopt the following criteria to measure the size of the forecasting errors of the volatility estimators.

$$RMSE = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\left(\hat{\sigma}(t+1) - \sigma_R(t+1)\right)^2}$$
$$MAE = \frac{1}{N}|\hat{\sigma}(t+1) - \sigma_R(t+1)|$$

where $RMSE$ is the root mean squared error of the volatility forecasts, MAE is the mean absolute error of the volatility forecasts, N is the number of observations, σ_R is the realized volatility until an option expiration date, and $\hat{\sigma}$ is the volatility forecast during the period between the current time and the option expiration date.

Using the methodologies above, a number of studies have attempted to answer the questions of which volatility has any explanatory power pertinent to future realized volatility, which outperforms the other, and which is an unbiased estimator. Examples of these studies include Latane and Rendleman (1976), Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Jorion (1995), Christensen and Prabhala (1998), Fleming (1998), Ederington and Guan (2002), and Blair, Poon, and Taylor (2001). According to the aforementioned studies, there is a strong consensus regarding the efficiency and informativeness of implied volatility compared to historical volatility. However, they do not adequately support its unbiasedness toward future realized volatility. These limited conclusions are consistent with the frequently observed disparities between implied volatility and ex-post realized volatility, which are denoted as "volatility spreads." (Bakshi and Madan, 2006; Kang, Kim and Yoon, 2010)

Given upon the apparent disparity between implied volatility and historical volatility, the natural next step is to figure out the reason for the bias of implied volatility from a theoretical viewpoint. Chernov (2007), using insight gained from option pricing literature, shows that ATM (at-the-money) implied volatility is an inefficient and biased forecast of future volatility. His main idea can be stated as follows: if additional risk such as volatility risk is not diversifiable and requires a premium, the implied volatility could be biased. Extensive empirical studies, including Benzoni (1998), Chernov and Ghysels (2000), Pan (2002), Jones (2003), and Eraker (2004), report a significant volatility risk premium and thus support Chernov's argument. Moreover, Bollerslev, Tauchen, and Zhou (2009), and Todorov (2010) measure these volatility spreads as the variance risk premium and analyze their information for future returns. Bakshi and Madan (2006) also examine the determinants of disparities between implied and realized volatilities in a different framework. Using the complete market model of Jackwerth (2000) to derive investors' risk preferences, they show that the disparity between two volatilities arises from a non-Gaussian property of returns and from the non-risk-neutrality of investors. More specifically, the volatility spreads are expressed as a function of investors' risk preferences and higher moments of return density through a parameterization technique based on Taylor series expansion. Two approaches of Chernov (2007) and Bakshi and Madan (2006) do not essentially differ from each other in terms of how they model the disparities. Given that volatility risk requires a premium unless an investor is risk-neutral and unless asset returns follow a Gaussian distribution, the two approaches ultimately suggest the same rationale for the determinant of the volatility spread, but through different ways.

Recently, Kang et al. (2010), based on the results of Bakshi and Madan (2006), modify the relation of volatility spreads by using risk-neutral moments rather than physical moments. This is an important modification in that it is easy to implement in empirical research. Moreover, it makes possible to develop a new volatility estimator, denoted as "adjusted implied volatility (AIV)", by modifying the implied volatility to incorporate both the non-risk neutrality of investors and non-Gaussian property of returns; consequently, it is consistent with the presence of additional risk premia. AIV has not only informational efficiency, but is also an unbiased estimator for future volatility. The superior performance of AIV guarantees the existence of risk premium associated with non-diversifiable risks (Kang et al., 2010).

In addition, the recent researches regarding volatility spreads bridge a gap between derivatives markets and underlying markets. Bollerslev et al. (2009) theoretically prove that the difference between implied and realized variances reflects the compensation for bearing variance risks that are not diversifiable, and denoted it as a variance risk premium. They show that the variance premium predicts short term future returns empirically. They use not only a beautiful analytic expression, but also assume the model non-parametrically, thereby making the model free from mis-specification errors. Similarly, Santa-Clara and Yan (2008) decompose a variance risk premium into volatility premium and jump premium, and highlight on the importance of jumps for predicting future returns. In particular, option-implied higher moments measure the jump intensity empirically. Similarly, Cremer and Weinbaum (2010) found that the put-call implied volatility spreads are informative on future individual stock return.

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