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ENTROPY DIMENSION AND VARIATIONAL PRINCIPLE

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ABSTRACT. Recently the notions of entropy dimension for topological dynamical system and measure theoretical dynamical system were introduced respectively. In this paper, we give a class of strictly ergodic models of which the topological entropy dimension ranges from zero to one and of which the measure theoretical entropy dimension is identically zero. Hence the variational principle on entropy dimension does not hold.

1. INTRODUCTION

Entropy is a notion of isomorphism invariant of dynamical system which measures the exponential growth rate of complexity. It is a complete invariant for the class of Bernoulli transformations. It is also well known that any measure theoretical dynamical system with positive entropy is isomorphic to a skew product of Bernoulli transformation with the same entropy.

Zero entropy systems have rich classes of transformations. Interval exchange transformations are zero entropy system. Simple maps in joining theory, which include Chacon transformation, irrational rotations and horocycle flows, also have zero entropy[8]. Indeed, the set of zero entropy systems is a dense G_δ subset of all homeomorphisms. To classify the zero entropy systems, we need some new invariants which are finer than entropy. There are several refinement notions of entropy[1, 2] and one of them is entropy dimension which has good properties. For example, two topological or measure thereotical dynamical system with different

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entropy dimension sets satisfying a certain condition are disjoint in the sense of Furstenberg[4, 5].

Let X be a compact metric space and T be a homeomorphism on X. The variational principle on the classical entropy says that the topological entropy of the dynamical system (X, T) is the supremum of all measure theoretical entropies of T. The aim of this paper is to show that the variational principle does not hold for entropy dimension. We consider a class of symbolic dynamical systems X_α's which is associated to the special kind of infinite sequence $\mathbf{u}_\alpha$ and show that these topological dynamical systems are uniquely ergodic and the measure theoretic entropy dimensions are identically zero. In [4], it was shown that the topological entropy dimension of the symbolic dynamical system X_α is α. Hence the variational principle does not hold for entropy dimension.

References

[1] S. Ferenczi, Measure-theoretic complexity of ergodic systems, *Israël J. Math*, **100** (1997), 189–207.

[2] A. Katok and J.-P. Thouvenot, Slow entropy type invariants and smooth realization of commuting measure-preserving transformations, *Ann. Inst. H. Poincaré Probab. Statist.*, **33** (1997), no. 3, 323–338.

[3] Julien Cassaigne, Constructing infinite words of intermediate complexity, Developments in language theory, 173–184, *Lecture Notes in Comput. Sci.*, **2450**, Springer, Berlin, 2003.

[4] Dou Dou, Wen Huang and Kyewon Koh Park, Entropy dimension of topological dynamics, submitted.

[5] Dou Dou, Wen Huang and Kyewon Koh Park, Entropy dimension of Measure-preserving systems, in preparation.

[6] D. Dou and K. Park, Examples of entropy generating sequence, submitted.

[7] Sébastien Ferenczi and Kyewon Koh Park, Entropy dimensions and a class of constructive examples, *Discrete Cont. Dyn. Syst.*, **17** (2007), no. 1, 133–141.

[8] E. Glasner, Ergodic Theory via Joinings, Mathematical Surveys and Monographs 101, American Mathematical Society, 2003.

[9] P. Walters, An introduction to ergodic theory. Graduate Texts in Mathematics 79, Springer-Verlag, New York-Berlin, 1982.

[10] S. Williams, Toeplitz minimal flows which are not uniquely ergodic, *Z. Wahrsch. Verw. Gebiete*, **67** (1984), no. 1, 95–107.

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