Trends in Mathematics - New Series
Information Center for Mathematical Sciences
Volume 13, Number 1, 2011, pages 7–13

BUILDING SIGNAL EXTRACTION MODEL BASED ON NEW OBJECTIVE FUNCTION

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ABSTRACT. This paper builds a signal extraction model of currency crisis prediction based on new objective function. The optimal threshold of early warning indicators minimizes the new objective function, more general than the noise-signal ratio which the current signal extraction model depends upon. Using composite crisis index and its associated cut-off probability the paper shows that the crisis forecast based on the general objective function performs better.

1. INTRODUCTION

The benefit of signal extraction model (SEM) as an early warning system (EWS) is simplicity and easiness although it has many flaws any kind of EWS does. Recently, researchers using SEM make effort to enhance tools for financial stability surveillance by developing a framework for assessing systemic risks stemming from domestic and global macro-financial vulnerabilities.[1]

Instead of the noise signal (NS) ratio this paper considers more general objective function. It is a function of noise and signal with the weight on them is not necessarily same. Thus, in this paper NS ratio is one special case such that the two weights are same. The optimal threshold computed from the new objective function enables to build EWS based on the different perception on future crises.

For example, when model A has more weight on noises than model B, the threshold level of an indicator in model A should be set tighter than model B. As a result, a warning issued by model A may not be issued by model B. Naturally the pattern of crisis prediction may not be same between the two models. Consequently, the runner effectively runs two different forecasting models based on the same data.

Assigning the objective function five different parameter values this paper is able to build five SEMs based on each parameter value. Applying the same cut-off probability of crisis associated with the composite crisis index computed from warnings of individual indicators the paper finds that the new objective function improves crisis prediction.

[1] For example, Lo Duca and Peltonen (2011).

2. Building signal extraction model

The signal extraction model has three main building blocks: crisis identification, choice of indicators and the way to generate crisis prediction.[2] Signaling window preceding the crisis is set 24 months. If, for instance, an indicator sends a signal that is followed by a crisis within 24 months it is labeled a good signal.[3] In-sample period is set from 1970.1 to 1995. 6 and out-sample period from 1995. 7 to 1997. 6. Given the signaling window the out-sample forecast covers period including East Asian crisis which first started in Thailand in 1997.7.

Crisis identification. Following common practice we define crisis index I_t which represents foreign exchange market pressure:

$$I_t = (\triangle e_t/e_t) - (\sigma_e/\sigma_R)(\triangle R_t/R_t)$$

where σ_e and σ_R are the standard deviation of the rate of change of exchange and reserves, respectively. The exchange rate t e is the unit of local currency in terms of one USD and reserves R_t is denoted also in USD. Thus, both depreciations of the exchange rate and declines in reserves raise the level of the index. Currency crisis is defined as an event where this index is more than 2.5 standard deviations above the mean.

$$\begin{aligned} \text{Crisis} &= 1 \quad \text{if } I_t > 2.5\sigma_{I_t} + \mu_{I_t} \\ &= 0 \quad \text{otherwise} \end{aligned}$$

where σ_{I_t} and μ_{I_t} are the sample standard deviation and the sample mean of the crisis index, respectively. 99 crises have been identified during 1970.1-1997.6 and 13 crises have been updated through 1997.7 to 1999.6.

Indicators. In SEM vulnerability to crisis is signaled when one or more indicator variables deviate significantly from its behavior during tranquil period. 15 monthly indicator variables popularly used in SEM are considered and they are listed in the appendix.[4] When the economy is vulnerable to currency crises the indicators are expected to alarm.

Objective Function. The outcome of an indicator is placed into one of cells A, B, C, or D in the following 2×2 table.

	Crisis occurs in the following 24 months	No crisis occurs in the following 24 months
Signal	A	B
No signal	C	D

[2]We cover 24 countries which previously experienced currency crises: Indonesia, Malaysia, S. Korea, Thailand, the Philippines, Argentina, Brazil, Bolivia, Chile, Colombia, Mexico, Peru, Uruguay, Venezuela, Denmark, Finland, Greece, Norway, Spain, Sweden, Egypt, S. Africa, Israel, Turkey.

[3]However, if crises follow a previous crisis within three months we regard them as the same crisis. The exclusion of windows is not to identify as new crises if they are just continuations of the previous ones.

[4]For example, Goldstein et al.(2000)

The paper consider the following objective function

$$\alpha \log\{B/(B+D)\} - (1-\alpha)\log\{1 - C/(A+C)\},\ 0 \le \alpha \le 1$$

Here, α denotes weight on logarithm of the probability of the type II error. The objective function is weighted average of logarithm of the noise and that of the inverse of the signal ratio. When $\alpha = 0.5$ minimizing the objective function is equivalent to minimizing NS ratio.[5] Therefore, the new objective function will bring exactly the same crisis prediction as NS ratio. When $\alpha < 0.5$, however, the objective function more concerns the risk of missing signals. When $\alpha > 0.5$, it has more weight on the risk of false alarms.

Furthermore, the weight α also affects the validity of indicators. Assigning α zero and up to one with increment of 0.1 the paper considers 11 parameter values. Within a range of $0 \le \alpha \le 0.3$ and $0.7 \le \alpha \le 1$ is the level of the optimal thresholds same and, therefore, it turns out that the same data set effectively generates five different SEMs.

Optimal Threshold. Table 1 reports the optimal threshold of 15 indicators which minimizes the objective function. For each value of α only indicators having NS ratio less than one is valid and the optimal threshold of each indicator is not necessarily same. This implies de facto five different SEMs are constructed based on the same data set.

TABLE 1. Performance of Indicators associated with α

	$0 \le \alpha \le 0.3$		$\alpha = 0.4$		$\alpha = 0.5$		$\alpha = 0.6$		$0.7 \le \alpha \le 1$	
	TV	NS	TV	NS	TV	NS	TV	NS	TV	NS
TOT	20	1.07	0.5	0.09	0.5	0.09	0.5	0.09	0.5	0.09
REX	80	0.40	88	0.29	99.5	0.18	99.5	0.18	99.5	0.18
M2/res.	80	0.66	80	0.66	99	0.43	99.5	0.43	99.5	0.43
Real output	20	0.90	20	0.90	1	0.53	0.5	0.53	0.5	0.53
D-F rate differential	80	1.03	82.5	0.95	99	0.56	99.5	0.64	99.5	0.64
Reserves	20	0.70	20	0.70	5	0.56	0.5	0.60	0.5	0.60
Real int. rate	80	0.95	81.5	0.91	99.5	0.57	99.5	0.57	99.5	0.57
Exports	20	0.72	19	0.71	1	0.63	0.5	0.65	0.5	0.65
Imports	80	1.14	80	1.14	99.5	0.66	99.5	0.66	99.5	0.66
Stock price	19.5	0.79	19.5	0.79	7	0.68	1.5	0.71	0.5	1.38
Ex. real M1	80	1.01	80	1.01	99.5	0.71	99.5	0.71	99.5	0.71
M2 multiplier	80	0.88	80	0.88	99	0.81	99.5	0.86	99.5	0.86
DC/GDP	80	0.90	80	0.90	85.5	0.85	99.5	1.80	99.5	1.80
Bank deposit	20	1.31	20	1.31	2	0.95	0.5	1.19	0.5	1.19
L-D rate	80	1.19	80	1.20	80.5	1.18	99.5	1.66	99.5	1.66

TV: optimal threshold level (%); The indicator in shaded area is invalid at the specific level of α.

[5]The decision rule to minimize the noise-to-signal ratio can also be put in the context of more standard statistical setting, using type I and type II errors. The size of a type I error is defined as the probability of rejecting the null hypothesis that a crisis occurs, that is, $C/(A+C)$. Similarly, the size of a type II error is the probability of not rejecting the null hypothesis when the hypothesis is false, $B/(B+D)$. Therefore, minimizing the noise-signal ratio is equivalent to minimizing the ratio of type II errors to one minus the ratio of type I errors.

Composite crisis index. In order to fix false alarms SEM has devised a composite indicator of vulnerability, so-called the composite crisis index which aggregates the information from the different variables into a single prediction. The composite crisis index I_t is defined as:

$$I_t = \sum S_t^j / \omega^j$$

where S_t^j is equal to one if indicator j crosses the threshold in period t and zero otherwise, and ω^j is the NS ratio of indicator j shown in Table 1. Therefore, the index weighs more heavily on the alarm issued by an indicator with lower NS ratio. The period t probability of crisis within 24 months given that I_t falls within the range I_a and I_b is computed using the following formula from in-sample panel data:

$$\begin{aligned}
&P(C_{t,t+24}|I_a < I_t < I_b) \\
=&\{\text{Months with } I_a < I_t < I_b \text{ given a crisis occurs within 24 months}\} \\
&/\{\text{Months with } I_a < I_t < I_b\}
\end{aligned}$$

Table 2 reports the probability of crises associated with the composite crisis index for five SEM.[6] From the definition of the composite crisis index it is clear that it is greater than one as long as any valid indicator issues a warning and zero, otherwise.

Table 2. Probability of crisis associated with composite crisis index

$0 \leq \alpha \leq 0.3$			$\alpha = 0.4$			$\alpha = 0.5$			$\alpha = 0.6$			$\alpha \geq 0.7$		
from	to	prob	from	to	prob	from	to	prob	from	to	prob	from	to	prob
0	1	0.18	0	1	0.19	0	1	0.24	0	1	0.25	0	1	0.25
1	2	0.22	1	2	0.24	1	2	0.30	1	2	0.32	1	2	0.32
2	3	0.26	2	3	0.22	2	3	0.37	2	3	0.18	2	3	0.13
3	4	0.34	3	4	0.29	3	4	0.49	3	4	0.46	3	4	0.46
4	5	0.34	4	5	0.38	4	5	0.31	4	5	NA	4	5	1.00
5	6	0.41	5	6	0.41	5	6	0.55	5	6	0.63	5	6	0.64
6	7	0.45	6	7	0.44	6	7	0.67	6	7	1.00	6	7	NA
7	8	0.61	7	8	0.51	7	8	0.71	7	8	0.71	7	8	0.67
8	9	0.31	8	9	0.75	8	9	0.75	8	9	NA	8	9	NA
9	10	0.87	9	10	0.75	9	10	1.00	9	10	NA	9	10	NA
10	11	0.88	10	11	0.33	10	11	NA	10	11	1.00	10	11	1.00
11	12	1.00	11	12	0.71	11	12	0.75	11	12	0.83	11	12	0.83
			12	13	1.00	12	13	1.00	12	13	1.00	12	13	1.00
			13	14	1.00									
			14	15	1.00									

Standard SEM with $\alpha = 0.5$ Table 3 reports the performance of crisis prediction using the cutoff probability picked up by Table 2 during both in-sample and out-sample period. Adjusting cut-off probability given certain level of α can refine predictions. However, an increase in the cut-off probability can reduce false alarms only at the cost of the missing signals. Furthermore, the trade-off between the two

[6]Unfortunately the relation is not monotonic. The probability of crisis falls to 0.31 in the range of 4-5. It is not because the probability actually falls, but because there are not enough number of crisis identified in the range.

types of error is very sharp. The number of correct calls in the last row drops dramatically.

Such sharp trade-off makes SEM unable to perform well. Not so good performance of SEM may reflect the inefficiency due to the forecasting rule the model presumes. Here, minimizing the noise-to-signal ratio is one such example. The optimal threshold minimizing NS ratio may not be necessarily 'optimal'. Although forecast by a single indicator should minimize NS ratio indicators *jointly* forecast crises. As shown in this section more general objective function is able to lead better prediction.

TABLE 3. Crisis prediction using cut-off probability

Cut off prob	A	B	C	D	Sum	T1	T2	NS	TC	CC
In-sample										
0.30	570	1178	1345	4239	7332	0.70	0.22	0.73	99	91
0.37	149	189	1766	5228	7332	0.92	0.03	0.45	99	60
0.49	97	99	1818	5318	7332	0.95	0.02	0.36	99	53
Out-sample										
0.30	85	112	92	287	576	0.52	0.28	0.58	17	17
0.37	37	23	140	376	576	0.79	0.06	0.28	17	14
0.49	26	14	151	385	576	0.85	0.04	0.24	17	8

T1: probability of type I error; T2: probability of type II error; TC: total number of crises identified; CC: number of crises correctly called

Based on various levels of α but applying the same cut-off probability the paper will assess the performance of the five models constructed in Table 2. The cut-off probability is set no less than 0.3 and it is 0.34 for $\alpha \leq 0.3$, 0.38 for $\alpha = 0.4$, 0.32 for $\alpha = 0.6$ and $\alpha \geq 0.7$. Table 4 reports the performance of the crisis prediction. As is expected the in-sample forecast of the five models are pretty consistent with the out-sample forecast. Particularly in-sample forecast for $\alpha = 0.3$ and 0.4 is superior to $\alpha = 0.5$ in every respect. The same is true for out-sample forecast except the number of crisis correctly called. It is because even though thresholds are set looser than $\alpha = 0.5$ alarms will be issued only when the composite crisis indicator is no less than that associated with the cut-off probability assigned.

It is interesting that crises tend to be commonly predicted in many models. All five models have predicted at least 2/3 of the total number of crises identified. The outright implication is that the seven indicators which commonly exist in the five models matter. On the other hand, the standard SEM with $\alpha = 0.5$ has issued too many false alarms. Probably it is due to the fact that the model has the largest number of valid indicators. As a result it has issued many noises and signals as well and based on NS ratio criterion the model performs worst. Even though it has predicted all 17 crises in out-sample forecast too many noises make the model hard to trust.

However, warnings commonly issued are not necessarily correct calls. It turns out that in Spain and Sweden alarms have been issued by all five models but turn out to be noises. Also models with $\alpha = 0.3$, 0.4 and $\alpha = 0.5$ have given false alarms to Argentina, Finland, Greece and Egypt. This forecasting error implies that SEM as an early warning system has some limits.

Table 4. Performance of Crisis Prediction: cut-off prob. ≥ 0.3

α	A	B	C	D	Sum	T1	T2	NS	TC	CC	PCS
In-sample											
[0, 0.3]	752	1159	1163	4258	7732	0.61	0.21	0.54	99	93	0.39
0.4	605	774	1310	4643	7332	0.68	0.14	0.45	99	92	0.44
0.5	570	1178	1345	4239	7332	0.70	0.22	0.73	99	91	0.33
0.6	151	251	1764	5166	7332	0.92	0.05	0.59	99	70	0.38
[0.7, 1]	141	225	1774	5192	7332	0.93	0.04	0.56	99	66	0.39
Out-sample											
[0, 0.3]	96	59	81	340	576	0.46	0.15	0.27	17	16	0.62
0.4	86	46	91	353	576	0.51	0.12	0.24	17	16	0.65
0.5	85	112	92	287	576	0.52	0.28	0.58	17	17	0.54
0.6	31	33	146	366	576	0.82	0.08	0.47	17	12	0.48
[0.7, 1]	29	32	148	367	576	0.84	0.08	0.49	17	12	0.48

T1: probability of type I error; T2: probability of type II error; PCS: conditional probability of crisis (PCS=A/(A+B)): TC: total number of crises identified; CC: number of crises correctly called

Or it may reflect the cause of the currency crisis. Warnings issued by indicators are identified as correct calls in East Asia but the same indicators deliver false alarms in Europe except Norway.[7] This forecast result has different implication from Zhuang and Dowling (2002) who use panel data of six Asian countries. They claim that weak fundamentals caused East Asian crisis based on the observation that alarming indicators such as real exchange rate, domestic credit, real output are barely related to self-fulfilling. However, their claim may not be justified in this paper and it comes into logical collision. Many indicators alarmed in Zhuang and Dowling also have given alarms and the same alarms turned out to be correct calls in East Asia but false alarms in Europe. This paper suggests that contrary to Zhuang and Dowling the contagion effect explains the East Asian crisis as Sachs, Tornell and Velasco (1996) proposes. That is, it is a signal that the economy is in crisis zone when these indicators alarm. But it is self-fulfilling prophecy or herd behavior such as investors panic which actually ignites crisis.

3. Concluding Remarks

This paper has proposed the new objective function other than NS ratio and built signal extraction model. NS ratio is a special form of the objective function considered. Depending on the specific value of the weight in the objective function five models have been built and executed out-sample forecast using cut-off probability no less than 0.3. Many crises have been predicted commonly in models based on the new objective function while some noises are also commonly found. In particular the standard model minimizing NS ratio has the most type II errors and the lowest conditional probability of crisis. Although not reported in this paper by extending sample period the paper confirms better prediction when the new objective function is used.

[7]For example, in Spain and Sweden real exchange rate indicator issued alarms 10 and 15 times when $\alpha \leq 0.3$ and five and 14 times when $\alpha = 0.4$. But all turns out noises.

Appendix

Indicator

Indicator	rate/level	Number of variables
Current account		
Terms of trade (TOT)	rate	874
Real exchange rate (REX)	level	6750
Exports (Exports)	r	6923
Imports (Imports)	r	6965
Capital account		
M2/ international reserves (M2/res.)	r	6992
Domestic and foreign real interest rate differential (D-F rate differential)	l	3775
Total reserves minus gold (Reserves)	r	7029
Real sector		
Real output(Real output)	r	5449
Real interest rate (Real int. rate)	l	4454
Stock prices (Stock price)	r	1751
Financial sector		
Ratio of lending interest rate to deposit interest rate (L-D rate)	l	3745
Excess real M1 balances (Ex. real M1)	l	6896
M2 multiplier (M2 multiplier)	r	6710
Domestic credit/ GDP (DC/GDP)	r	6725
Bank deposit (Bank deposit)	r	6581
Sum		81619

These indicators are available from *World Development Indicators*, World Bank, *International Financial Statistics*, IMF.

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