APPROXIMATION-SOLVABILITY OF
A CLASS OF $A$-MONOTONE VARIATIONAL
INCLUSION PROBLEMS

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ABSTRACT. First the notion of the $A$-monotonicity is applied to the approximation-solvability of a class of nonlinear variational inclusion problems, and then the convergence analysis is given based on a projection-like method. Results generalize nonlinear variational inclusions involving $H$-monotone mappings in the Hilbert space setting.

1. Introduction and Preliminaries

Based on the notion of the $A$-monotonicity, recently the author [8] studied a new class of variational inclusion problems, including hemivariational inclusion problems applied to engineering and mechanics. The obtained results generalize some variational inclusion problems introduced and studied by Fang and Huang [2]. They solved nonlinear variational problems applying the resolvent operator technique. These notions have energized the theory of maximal monotone mappings in general. In this paper consider applications of $A$-monotone mappings to the approximation-solvability of a class of nonlinear variational inclusions in a Hilbert space setting. The convergence analysis for the solution is based on a projection-like method. The obtained results generalize results on general maximal monotone and $H$-monotone mappings, including [2]. We have established some auxiliary results as well. For more details on the generalized monotonicity, we recommend [1-9].

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Definition 1. [8] Let $A: X \to X^*$ be a mapping from a reflexive Banach space $X$ into its dual $X^*$ and $M: X \to \mathcal{P}(X^*)$ be another mapping from $X$ into the power set $\mathcal{P}(X^*)$ of $X^*$. The map $M$ is said to be $A$-monotone if $M$ is $m$-relaxed monotone and $A + \rho M$ is maximal monotone for $\rho > 0$.

Definition 2. [2] Let $H: H \to H$ and $M: H \to 2^H$ be any two mappings on $H$. The map $M$ is said to be $H$-monotone if $M$ is monotone and $(H + \rho M)(H) = H$ holds for $\rho > 0$.

This is equivalent to stating that $H + \rho M$ is maximal monotone if $M$ is monotone and $H + \rho M$ is maximal monotone. If $H$ is strictly monotone and $M$ is $H$-monotone, then $M$ is maximal monotone.

Let the resolvent operator $J_{H,M}^\rho: H \to H$ be defined by

$$J_{H,M}^\rho(u) = (H + \rho M)^{-1}(u) \quad \forall \ u \in H.$$

On the top of that, if $H$ is $r$-strongly monotone and $M$ is $H$-monotone, then the resolvent operator $J_{H,M}^\rho$ is $(1/r)$-Lipschitz continuous for $r > 0$. From now on, $\mathcal{P}(H)$ shall denote the power set $2^H$.

Definition 3. A mapping $T: H \to H$ is said to be:

(i) $r$-strongly monotone with respect to $A$ if there exists a positive constant $r$ such that

$$\langle T(x) - T(y), A(x) - A(y) \rangle \geq ||x - y||^2 \quad \forall \ x, y \in H.$$

(ii) $r$-strongly monotone if there exists a positive constant $r$ such that

$$\langle T(x) - T(y), x - y \rangle \geq r \ ||x - y||^2 \quad \forall \ x, y \in H.$$

(iii) $m$-relaxed monotone if there is a positive constant $m$ such that

$$\langle T(x) - T(y), x - y \rangle \geq (-m) ||x - y||^2 \quad \forall \ x, y \in H.$$

(iv) $(\gamma, s)$-relaxed cocoercive with respect to $A$ if there exist positive constants $\gamma$ and $s$ such that

$$\langle T(x) - T(y), A(x) - A(y) \rangle \geq (-\gamma) ||T(x) - T(y)||^2 + s \ ||x - y||^2 \quad \forall \ x, y \in H.$$
Lemma 1. Let \( A: H \to H \) be \( r \)-strongly monotone and \( M: H \to P(H) \) be \( A \)-monotone. Then the resolvent operator \( J^\rho_{A,M}(u): H \to H \) is \([1/(r - \rho m)]\)-Lipschitz continuous for \( 0 < \rho < r/m \), where \( r, \rho \) and \( m \) are positive constants.

Proof. For any \( u, v \in H \), we have from the definition of the resolvent operator that

\[
J^\rho_{A,M}(u) = (A + \rho M)^{-1}(u)
\]

\[
J^\rho_{A,M}(v) = (A + \rho M)^{-1}(v).
\]

It follows that

\[
(1/\rho)[u - A(J^\rho_{A,M}(u))] \in M(J^\rho_{A,M}(u))
\]

\[
(1/\rho)[v - A(J^\rho_{A,M}(v))] \in M(J^\rho_{A,M}(v)).
\]

Since \( M \) is \( A \)-monotone (and hence \( m \)-relaxed monotone), it implies that

\[
(1/\rho)<u - A(J^\rho_{A,M}(u)) - [v - A(J^\rho_{A,M}(v))], J^\rho_{A,M}(u) - J^\rho_{A,M}(v)> \geq (-m)\| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \|^2.
\]

As a result, we have

\[
\| u - v \| \| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \| \geq <u - v, J^\rho_{A,M}(u) - J^\rho_{A,M}(v)>
\]

\[
\geq <A(J^\rho_{A,M}(u)) - A(J^\rho_{A,M}(v)), J^\rho_{A,M}(u) - J^\rho_{A,M}(v)>
\]

\[
- \rho m \| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \|^2
\]

\[
\geq r \| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \|^2 - \rho m \| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \|^2
\]

\[
= (r - \rho m) \| J^\rho_{A,M}(u) - J^\rho_{A,M}(v) \|^2.
\]
Lemma 2. Let $M: H \to P(H)$ be $A$-monotone. Then the resolvent operator $J_{\rho}^{H,M}(u) := (I + \rho M)^{-1}: H \to H$ is $[1/(1 - \rho m)]$-Lipschitz continuous for $0 < \rho < 1/m$, where $\rho$ and $m$ are positive constants and $I$ is the identity mapping.

Lemma 3. Let $H: H \to H$ be $r$-strongly monotone and $M: H \to P(H)$ be $H$-monotone. Then the resolvent operator $J_{\rho}^{H,M}(u): H \to H$ is $r$-cocoercive.

Proof. For any $u, v \in H$, we have from the definition of the resolvent operator that

$$J_{\rho}^{H,M}(u) = (H + \rho M)^{-1}(u)$$

$$J_{\rho}^{H,M}(v) = (H + \rho M)^{-1}(v).$$

It follows that

$$(1/\rho)[u - H(J_{\rho}^{H,M}(u))] \in M(J_{\rho}^{H,M}(u))$$

$$(1/\rho)[v - H(J_{\rho}^{H,M}(v))] \in M(J_{\rho}^{H,M}(v)).$$

Since $M$ is $H$-monotone and $H$ is $r$-strongly monotone, it implies that

$$< u - v, J_{\rho}^{H,M}(u) - J_{\rho}^{H,M}(v)> \geq 0.$$

As a result, we have

$$< u - v, J_{\rho}^{H,M}(u) - J_{\rho}^{H,M}(v)> \geq r ||J_{\rho}^{H,M}(u) - J_{\rho}^{H,M}(v)||^2.$$

For $H = I$ and $r \leq 1$, $J_{\rho}^{H,M}(u) = (I + \rho M)^{-1}: H \to H$ is 1-cocoercive.

Lemma 4. [2] Let $H: H \to H$ be $r$-strongly monotone and $M: H \to P(H)$ be $H$-monotone. Then the resolvent operator $J_{\rho}^{H,M}: H \to H$ is $(1/r)$-Lipschitz continuous for a positive constant $r$.

Lemma 5. Let $A: H \to H$ be $r$-strongly monotone and $M: H \to P(H)$ be $A$-monotone. Then the resolvent operator $J_{\rho}^{A,M}: H \to H$ is $(r - \rho m)$-cocoercive for $0 < \rho < r/m$, where $r, \rho$ and $m$ are positive constants.

Proof. For any $u, v \in H$, we have from the definition of the resolvent operator that
\[ J_{\rho, A, M}(u) = (A + \rho M)^{-1}(u) \]

\[ J_{\rho, A, M}(v) = (A + \rho M)^{-1}(v). \]

It follows that

\[ \frac{1}{\rho}[u - A(J_{\rho, A, M}(u))] \in M(J_{\rho, A, M}(u)) \]

\[ \frac{1}{\rho}[v - A(J_{\rho, A, M}(v))] \in M(J_{\rho, A, M}(v)). \]

Since \( M \) is \( A \)-monotone (and hence \( m \)-relaxed monotone), it implies that

\[ \frac{1}{\rho} [u - A(J_{\rho, A, M}(u)) - v + A(J_{\rho, A, M}(v))] \geq (-m) \| J_{\rho, A, M}(u) - J_{\rho, A, M}(v) \|^2. \]

As a result, we have

\[ < u - v, J_{\rho, A, M}(u) - J_{\rho, A, M}(v)> \geq < A(J_{\rho, A, M}(u)) - A(J_{\rho, A, M}(v)), J_{\rho, A, M}(u) - J_{\rho, A, M}(v)> \]

\[ - \rho m \| J_{\rho, A, M}(u) - J_{\rho, A, M}(v) \|^2 \]

\[ \geq r \| J_{\rho, A, M}(u) - J_{\rho, A, M}(v) \|^2 - \rho m \| J_{\rho, A, M}(u) - J_{\rho, A, M}(v) \|^2 \]

\[ = (r - \rho m) \| J_{\rho, A, M}(u) - J_{\rho, A, M}(v) \|^2. \]

Example 1. [3, Lemma 7.11] Let \( X \) be a reflexive Banach space and \( X^* \) its dual. Suppose that \( f: X \to \mathbb{R} \) is \( m \)-strongly monotone and \( f: X \to \mathbb{R} \) is locally Lipschitz such that \( \partial f \) is \( \alpha \)-relaxed monotone. Then \( \partial f \) is \( A \)-monotone (i.e., \( A + \partial f \) is maximal monotone for \( m - \alpha > 0 \), where \( m, \alpha > 0 \)) for \( \rho = 1 \). Since \( A \) is \( m \)-strongly monotone and \( \partial f \) is \( \alpha \)-relaxed monotone, it implies that \( A + \partial f \) is \( (m - \alpha) \)-strongly monotone. It further follows that \( A + \partial f \) is pseudomonotone one and hence \( A + \partial f \) is, in fact, maximal monotone.

Example 2. [5, Theorem 4.1] Let \( X \) be a reflexive Banach space and \( X^* \) its dual. Let \( A: X \to X^* \) be \( a \)-strongly monotone and \( B: X \to X^* \) be \( c \)-strongly Lipschitz continuous. Let \( f: X \to \mathbb{R} \) be locally Lipschitz such that \( \partial f \) is relaxed \( \alpha \)-monotone. Then \( \partial f \) is \( (A - B) \)-monotone (i.e., \( A - B + \partial f \) is maximal monotone for \( -c - \alpha > 0 \)) for \( \rho = 1 \).
Let $H$ be a real Hilbert space and let $A$ be a nonempty closed convex subset of $H$. Let $T : H \to H$ be a nonlinear mapping. Let $A : H \to H$ and $M : H \to \mathcal{P}(H)$ be any mappings. Then the problem of finding $a \in H$ such that

$$0 \in T(a) + M(a)$$

is called the nonlinear variational inclusion (NVI) problem.

Let $f : H \to \mathbb{R}$ be a locally Lipschitz continuous function and $\partial f : H \to \mathcal{P}(H)$ be $m$-relaxed monotone. Then for $M = \partial f$, the NVI (1) problem reduces to: find an element $a \in H$ such that

$$0 \in T(a) + \partial f(a).$$

If $f : H \to \mathbb{R}$ is proper, convex and lower semicontinuous, and $f'(x)$ denotes the gradient of $f$ at $x$ such that $M(x) = \partial f(x)$ for all $x \in H$, then problem (1) reduces to: find an element $a \in A$ such that

$$< T(a), x - a > + < f'(a), x - a > \geq 0 \quad \forall \ x \in H, \quad (3)$$

where $A$ is a nonempty closed convex subset of $H$.

It follows from (3) that

$$< T(a), x - a > + f(x) - f(a) > \geq 0 \quad \forall \ x \in H. \quad (4)$$

When $M(x) = \partial A(x)$ for all $x \in A$, where $A$ is a nonempty closed convex subset of $H$ and $\partial A$ denotes the indicator function of $A$, the NVI (1) problem reduces to the problem: determine an element $a \in A$ such that

$$< T(a), x - a > \geq 0 \quad \forall \ x \in A. \quad (5)$$

Let $f : H \to \mathbb{R} \cup \{\pm \infty\}$ be a functional on $H$. A functional $x^* \in H$ is a subgradient of $f$ at $u$ iff

$$f(u) \neq \mp \infty \text{ and } f(v) \geq f(u) + < x^*, v - u > \quad \forall \ v \in H.$$

The set of all subgradients of $f$ at $u$, denoted $\partial f(u)$, is called the subdifferential at $u$. If there exists no subgradients, then $\partial f(u) = \emptyset$.

A function $f : H \to \mathbb{R} \cup \{\pm \infty\}$ is said to be one-sided directional Gâteaux-differentiable at $x^*$ if there is the $f'(x^*, h)$ such that
\[
\lim_{\mu \to 0} \left[ f(x^* + \mu h) - f(x^*) \right]/\mu = f'(x^*, h) \quad \forall h \in H.
\]

If \( f \) is convex, then \( f \) is one-sided directional Gâteaux-differentiable at every point \( x \in H \) with \( h f(x) \neq \mp \infty \). On the top of that, we have

\[
f(x) - f(u) \geq f'(u, x - u) \quad \forall x \in H,
\]

and

\[
f'(u, x - u) \geq - f'(u, -(x - u)) \quad \forall x \in H.
\]

A function \( f: H \to \mathbb{R} \cup \{\pm \infty\} \) is called locally Lipschitz at \( x \) if a neighborhood \( U \) of \( x \) exists such that \( f \) is finite on \( U \) and

\[
\left| f(x) - f(y) \right| \leq c \|x - y\| \quad \forall x \in H,
\]

where \( c \) is a positive constant depending on \( U \).

Next we define the generalized directional differential (in the sense of Clarke) of \( f \) at \( x \) in the direction \( y \), denoted \( f^\delta(x, y) \), by

\[
\lim_{\mu \to 0^+, h \to 0} \frac{[f(x + h + \mu y) - f(x + h)]}{\mu} = f^\delta(x, y).
\]

The corresponding generalized gradient of \( f \) at \( x \), denoted by \( \partial f(x) \), is defined by

\[
\partial f(x) = \{x^*: x^* \in H, f^\delta(x, y - x) \geq < x^*, y - x > \quad \forall y \in H\},
\]

where \( \partial f: H \to 2^H \). If we set \( M(x) = \partial f(x) \) in (1), then it reduces to a constrained problem: find an element \( a \in H \) such that

\[
<T(a), x - a > + f^\delta(a, x - a) \geq 0 \quad \forall x \in H, \tag{6}
\]

Let \( B(u_0, r) \) denote the closed ball in \( H \) defined by

\[
B(u_0, r) = \{v \in H: \|u_0 - v\| \leq r \text{ for } r > 0\},
\]

where \( u_0 \) is the center and \( r \) is the radius. Let \( A \) be a closed and star-shaped subset of \( H \) with respect to \( B(u_0, r) \). \( A \) is star-shaped with respect to \( B(u_0, r) \) if

\[
v \in A \iff \lambda v + (1 - \lambda)w \in A \text{ for any } \lambda \in [0, 1] \text{ and } w \in B(u_0, r).
\]

Let \( d_\lambda : H \to \mathbb{R} \) denote the distance function of \( A \) defined by
Further more, let $T_A(u)$ denote Clarke’s tangent cone of $A$ at $u$, which is defined by

$$T_A(u) = \{ k \in H : \forall u_n \to u, \ u_n \in A, \ \forall \lambda_n \to 0, \ \text{there exists} \ k_n \to k \ \text{such that} \ u_n + \lambda_n k_n \in A \}.$$

Note that $T_A(u)$ is a closed convex cone and it always contains zero. Now if we set $M(x) = \partial \delta(T_A(x))$, where $\delta(T_A)$ denotes the indicator function of $T_A(x)$, then the NVI (1) problem reduces to: find an element $a \in A$ such that

$$< T(a), k > \geq 0 \ \forall k \in T_A(a).$$

(7)

Since $A$ is not convex, the problem (7) is called a constrained hemivariational inequality (N HI) problem. Clearly, the NHI (7) problem reduces to the NVI (5) problem when $A$ is convex.

**Lemma 6.** Let $H$ be a real Hilbert space, let $A: H \to H$ be strictly monotone, and $M: H \to 2^H$ be $A$-monotone. Then an element $a \in H$ is a solution to the NVI (1) problem iff $a$ satisfies

$$a = J^\rho_{A,M}[A(a) - \rho T(a)],$$

(8)

where $T: H \to H$ is any mapping on $H$ and $\rho$ is a positive constant.

**Theorem 1.** Let $H$ be a real Hilbert space. Let $A: H \to H$ be $r$-strongly monotone and $\alpha$-Lipschitz continuous. Let $M: H \to P(H)$ be $A$-monotone. Suppose that $T: H \to H$ is a mapping such that $T$ is $(s)$-strongly monotone with respect to $A$ and $\mu$-Lipschitz continuous. If, in addition, there exists a constant $\rho > 0$ such that

$$\sqrt{\alpha^2 2ps + \rho^2 \mu^2} < r - \rho \mu,$$

then the NVI (1) problem has a unique solution.

**Proof.** For $u, v \in H$, let us define a mapping $\Lambda: H \to H$ by

$$\Lambda(u) = J^\rho_{A,M}(A(u) - \rho T(u)).$$

Then we have

$$\|\Lambda(u) - \Lambda(v)\| = \|J^\rho_{A,M}(A(u) - \rho T(u)) - J^\rho_{A,M}(A(v) - \rho T(v))\|$$

$$\leq \left[ \frac{1}{(r - \rho \mu)} \right] \| (A(u) - \rho T(u)) - (A(v) - \rho T(v)) \|. $$
It follows that
\[ \| A(u) - A(v) - \rho (T(u) - T(v)) \|^2 = \| A(u) - A(v) \|^2 + \rho^2 \| T(u) - T(v) \|^2 - 2\rho \langle A(u) - A(v), T(u) - T(v) \rangle \]
\[ \leq \alpha^2 \| u - v \|^2 + \rho^2 \mu^2 \| u - v \|^2 - 2\rho \| u - v \|^2 + 2\rho \gamma \| T(u) - T(v) \|^2 \]
\[ = (\alpha^2 - 2\rho \| u - v \|) \| u - v \|^2 . \]
Hence,
\[ \Lambda(u) - \Lambda(v) \leq \frac{\theta}{r - \rho m} \| u - v \| , \]
where \( \theta = \sqrt{\alpha^2 - 2\rho \mu^2} < r - \rho m \).

Hence, \( \Lambda: H \to H \) is a contraction for \( 0 < \rho < r/m \). This implies that there exists a unique element \( a \in H \) such that
\[ \Lambda(a) = a, \]
that means,
\[ a = J_{\rho A, M}^\rho (A(a) - \rho T(a)). \]
It follows from Lemma 6 that \( a \) is a unique solution to the NVI (1) problem.

**Corollary 1.** Let \( H \) be a real Hilbert space. Let \( A: H \to H \) be \( r \)-strongly monotone and \( \alpha \)-Lipschitz continuous. Let \( \partial f: H \to P(H) \) be \( A \)-monotone. Suppose that \( T: H \to H \) is \( (s) \)-strongly monotone with respect to \( A \) and \( \mu \)-Lipschitz continuous. If, in addition, there exists a constant \( \rho > 0 \) such that
\[ \sqrt{\alpha^2 - 2\rho \mu^2} < r - \rho m, \]
then the NVI (2) problem has a unique solution.

**Corollary 2.** Let \( H \) be a real Hilbert space. Let \( H: H \to H \) be \( r \)-strongly monotone and \( \alpha \)-Lipschitz continuous. Let \( M: H \to P(H) \) be \( H \)-monotone. Suppose that \( T: H \to H \) is a mapping such that \( T \) is \( (s) \)-strongly monotone with respect to \( H \) and \( \mu \)-Lipschitz continuous. If, in addition, there exists a constant \( \rho > 0 \) such that
\[
\sqrt{\alpha^2 2ps + \rho^2 \mu^2} < r,
\]
then the NVI (1) problem has a unique solution.

2. Convergence Analysis

In this section, we apply a projection-type iterative algorithm to approximate the unique solution to the NVI (1) problem.

**Algorithm 1.** For an arbitrarily chosen initial point \( a^0 \in H \), compute the sequence \( \{a^k\} \) such that

\[
a^{k+1} = (1 - \alpha^k) a^k + \alpha^k \mathcal{J}_{\mathcal{P}_{A,M}}[A(a^k) - \rho T(a^k)] \quad \text{for } k \geq 0,
\]

where the sequence \( \{\alpha^k\} \) satisfies

\[
0 \leq \alpha^k < 1 \quad \text{and} \quad \sum_{k=0}^{\infty} \alpha^k = \infty.
\]

**Theorem 2.** Let \( H \) be a real Hilbert space. Let \( A: H \rightarrow H \) be \( r \)-strongly monotone with respect to \( A \) and \( \alpha \)-Lipschitz continuous. Let \( M: H \rightarrow \mathcal{P}(H) \) be \( A \)-monotone. Suppose that \( T: H \rightarrow H \) is a mapping such that \( T \) is \((s)\)-strongly monotone with respect to \( A \) and \( \mu \)-Lipschitz continuous. If, in addition, there exists a constant \( \rho > 0 \) such that

\[
\sqrt{\alpha^2 2ps + \rho^2 \mu^2} < r - \rho m \quad \text{for} \quad \rho < r/m,
\]

and the sequence \( \{a^k\} \) is generated by Algorithm 1, then the sequence \( \{a^k\} \) converges to a unique solution to the NVI (1) problem.

Proof. Since in Theorem 1, it is shown that an element \( a \in H \) is the unique solution to the NVI (1) problem, we have

\[
\|[a^{k+1}] - a\| = \| (1 - \alpha^k) a^k + \alpha^k \mathcal{J}_{\mathcal{P}_{A,M}}[A(a^k) - \rho T(a^k)] - (1 - \alpha^k) a - \alpha^k \mathcal{J}_{\mathcal{P}_{A,M}}[A(a) - \rho T(a)] \|
\]

\[
\leq (1 - \alpha^k) \|[a^k] - a\| + \alpha^k \|[\mathcal{J}_{\mathcal{P}_{A,M}}[A(a^k) - \rho T(a^k)] - \mathcal{J}_{\mathcal{P}_{A,M}}[A(a) - \rho T(a)]\|
\]

\[
\leq (1 - \alpha^k) \|[a^k] - a\| + \alpha^k / (r - \rho m) \|[A(a^k) - A(a) - \rho (T(a^k) - T(a))]\|
\]

\[
\leq (1 - \alpha^k) \|[a^k] - a\| + \alpha^k / (r - \rho m) \sqrt{\alpha^2 2ps + \rho^2 \mu^2} \|[a^k] - a\|
\]
\[
\begin{align*}
&= \{1 - \alpha^k + [\alpha^k/(r - \rho m)]\theta\} \| (a^k - a) \| \\
&= [1 - \alpha^k + (\alpha^k/(r - \rho m))\theta] \| (a^k - a) \| \\
&= \{1 -(1 - \Theta)\alpha^k\} \| (a^k - a) \| \\
&\leq \Pi_{j=0}^k \{1 -(1 - \Theta)\alpha^j\} \| (a^0 - a) \|,
\end{align*}
\]

where $\Theta < 1$ for $\Theta = \theta/(r - \rho m)$ and for

\[
\sqrt{\alpha^2 ps + \rho^2 \mu^2} < r - \rho m.
\]

Since $\Theta < 1$ and $\sum_{k=0}^{\infty} \alpha^k$ is divergent, it implies from [9] that

\[
\lim_{k \to \infty} \Pi_{j=0}^k \{1 -(1 - \Theta)\alpha^j\} = 0.
\]

Now it follows from (5) that the sequence $\{a^k\}$ converges to $a$, the unique solution to the NVI (1) problem.

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