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DYNAMICS OF THE g-NAVIER-STOKES EQUATIONS

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ABSTRACT. The 2D g-Navier-Stokes equations have the following form,

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f},$$

with the continuity equation

$$\nabla\cdot(g\mathbf{u}) = 0,$$

where g is a suitable smooth real valued function. In this paper, we will give short remark of the derivation of the 2D g-Navier-Stokes equations from the 3D Navier-Stokes equations. Then, we will give the results of the g-Navier-Stokes equations in the view of the dynamical systems.

1. INTRODUCTION

The g-Navier-Stokes equations in spatial dimension 2 are a variation of the standard Navier-Stokes equations, and they assume the form,

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \quad \text{in} \quad \Omega, \tag{1.1}$$

$$\frac{1}{g}(\nabla\cdot g\mathbf{u}) = \frac{\nabla g}{g}\cdot\mathbf{u} + \nabla\cdot\mathbf{u} = 0, \quad \text{in} \quad \Omega, \tag{1.2}$$

where $g = g(x_1, x_2)$ is a suitable smooth real valued function defined on $(x_1, x_2) \in \Omega$ and Ω is a suitable bounded domain in R^2. Notice that if $g(x_1, x_2) = 1$, then the equation (1.1) and (1.2) reduce to the standard Navier-Stokes equations,

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \quad \text{in} \quad \Omega, \tag{1.3}$$

$$\nabla\cdot\mathbf{u} = 0, \quad \text{in} \quad \Omega. \tag{1.4}$$

While the g-Navier-Stokes equations form a meaningful problem in a 3-dimensional spatial region $\Omega \subset \mathbf{R}^3$, where $g = g(x_1, x_2, x_3)$ and $(x_1, x_2, x_3) \in \Omega$, we are specially interested in the 2-dimensional problem here. The reason for this is that the 2-dimensional g-Navier-Stokes equations arise in a natural way in the study of a standard 3-dimensional problem, as we show in the next section. We do not claim that the g-Navier-Stokes equations form a model of any fluid flow. They may, or may not. That they are derived from a standard 3-dimensional problem is the basis for our study.

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Before we present the derivation of the g-Navier-Stokes equations, it is convenient to recall some relevant aspects of the classical theory of the Navier-Stokes equations. For many years, the Navier-Stokes equations were investigated by many authors and the existence of the attractors for 2D Navier-Stokes equations was first proved by Ladyzhenskaya[3] and independently by Foias and Temam[2]. The finite dimensional property of the global attractor for general dissipative equations was first proved by Mallet-Paret[5] and Mañé[6]. For the analysis on the Navier–Stokes equations, one can refer to [1], [4], [9] and [10], specially [11] for the periodic boundary conditions.

In this paper, we will have the following organization. In section 2, we will present the derivation of 2D g-Navier-Stokes equations from 3D Navier-Stokes equations without the proofs(see Roh[7] for the details). In section 3, we will present main results of the g-Navier-Stokes equations in the view of the dynamical systems, eventhough one can find more results for the analysis on the g-Navier-Stokes equations from the author's works.

2. Derivation of the 2D g-Navier-Stokes equations

Let $\Omega_g = \Omega_2 \times [0, g]$, where Ω_2 is a bounded region in the plane and $g = g(x_1, x_2)$ is a smooth function defined on Ω_2 with $0 < m \le g(x_1, x_2) \le M$, for $(x_1, x_2) \in \Omega_2$. Now, we consider the $3D$ Navier-Stokes equations,

$$\begin{aligned} \frac{\partial \mathbf{U}}{\partial t} - \nu \Delta \mathbf{U} + (\mathbf{U} \cdot \nabla)\mathbf{U} + \nabla \Phi &= \mathbf{F}, \text{ in } \Omega_g \\ \nabla \cdot \mathbf{U} &= 0, \text{ in } \Omega_g, \end{aligned}$$

with the boundary condition

$$\mathbf{U} \cdot \mathbf{n} = 0 \quad \text{on} \quad \partial_{top}\Omega_g \cup \partial_{bottom}\Omega_g \tag{2.1}$$

where

$$\begin{aligned} \partial_{top}\Omega_g &= \{(x_1, x_2, x_3) \in \Omega_g \; : \; x_3 = g(x_1, x_2)\}, \\ \partial_{bottom}\Omega_g &= \{(x_1, x_2, x_3) \in \Omega_g \; : \; x_3 = 0\}. \end{aligned}$$

The lateral boundary condition corresponding to $\partial\Omega_2$ does not affect to the derivation of the 2D g-Navier-Stokes equations. But, in this paper we will consider the periodic boundary conditions to study the 2D g-Navier-Stokes equations.

Now we define $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2)$ as

$$\mathbf{u}_i = \mathbf{u}_i(x_1, x_2) = \frac{1}{g(x_1, x_2)} \int_0^{g(x_1, x_2)} \mathbf{U}_i(x_1, x_2, x_3) \, dx_3,$$

for $i = 1, 2$ and we get the following lemma.

Lemma 2.1. *Assume that $\nabla \cdot \mathbf{U} = 0$ in Ω_g and that (2.1) is valid. Then one has*

$$\nabla_2 \cdot (g\mathbf{u}) = \frac{\partial (g\mathbf{u}_1)}{\partial x_1} + \frac{\partial (g\mathbf{u}_2)}{\partial x_2} = \nabla g \cdot \mathbf{u} + g \; (\nabla_2 \cdot \mathbf{u}) = 0 \quad \text{in } \Omega_2,$$

where $\nabla_2 = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})$ and $\nabla g = (\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2})$.

Proof. See Roh[7] for the details. □

Now, we consider the special case like

$$\mathbf{U}(x_1, x_2, x_3) = (\mathbf{U}_1(x_1, x_2), \mathbf{U}_2(x_1, x_2), \mathbf{U}_3(x_1, x_2, x_3)).$$

By the previous lemma, for $\mathbf{u} = (U_1, U_2)$, one has $\nabla \cdot (g\mathbf{u}) = 0$ and $\mathbf{u}$ satisfies the 2D g-Navier-Stokes equations. Moreover, we have

$$\mathbf{U}_3(x_1, x_2, x_3) = -x_3 \ (\frac{\partial \mathbf{u}_1}{\partial x_1} + \frac{\partial \mathbf{u}_2}{\partial x_2}) = -x_3 \ (\nabla_2 \cdot \mathbf{u})$$

when (2.1) and $\nabla \cdot \mathbf{U} = 0$ in Ω_g are valid. This is the basis for our study of the 2D g-Navier-Stokes equations.

3. Main Results

Here, we consider the periodic boundary conditions on the domain $\Omega = (0,1) \times (0,1)$ and assume $\mathbf{u}, p$ and the first derivatives of $\mathbf{u}$ to be spatially periodic, i.e.,

$$\mathbf{u}(x_1 + 1, x_2) = \mathbf{u}(x_1, x_2) = \mathbf{u}(x_1, x_2 + 1), \quad (x_1, x_2) \in \mathbf{R}^2$$

and similarly for p and $\frac{\partial \mathbf{u}_i}{\partial x_j}$.

For the function g, throughout this paper, we assume that

(1) $g(\mathbf{x}) \in C^{\infty}_{per}(\Omega)$ and
(2) $0 < m \le g(x, y) \le M$, for all $(x, y) \in \Omega$.

Note that the constant function $g = 1$ is also included for our function g.

Now, we define the Hilbert space $L^2(\Omega, g) = L^2(\Omega, R^2, g)$, which is the space $L^2(\Omega)$ with the scalar product and the norm given by

$$< \mathbf{u}, \mathbf{v} >_g = \int_\Omega (\mathbf{u} \cdot \mathbf{v}) \ g \ d\mathbf{x} \quad \text{ and } \quad \| \mathbf{u} \|^{\mathbf{2}}_{\mathbf{g}} = < \mathbf{u}, \mathbf{u} >_{\mathbf{g}},$$

where $\mathbf{x} = (x_1, x_2)$. Similarly, we define the space $H^1(\Omega, g)$ which is the space $H^1(\Omega)$ with the norm by

$$\| \mathbf{u} \|_{H^1(\Omega,g)} = [\langle \mathbf{u}, \mathbf{u} \rangle_g + \sum_{i=1}^{2} \langle D_i \mathbf{u}, D_i \mathbf{u} \rangle_g]^{\frac{1}{2}},$$

where $\frac{\partial \mathbf{u}}{\partial x_i} = D_i \mathbf{u}$. Specially, for the constant function $g = 1$, we denote that

$$\| \mathbf{u} \|_1 = \| \mathbf{u} \|, \quad \| \mathbf{u} \|_{H^1(\Omega,1)} = \| \mathbf{u} \|_{H^1(\Omega)}.$$

One can see easily that the norm $\| \mathbf{u} \|$ is equivalent to the norm $\| \mathbf{u} \|_g$ as well as the norm $\| \mathbf{u} \|_{H^1(\Omega)}$ is equivalent to the norm $\| \mathbf{u} \|_{H^1(\Omega,g)}$.

Now, we consider the following closed subspaces of $L^2(\Omega, g)$;

$$\tilde{H} = CL_{L^2(\Omega,g)} \{ \mathbf{u} \in C^{\infty}_{per}(\Omega) \ : \ \nabla \cdot g\mathbf{u} = 0 \}.$$

Then, we define the orthogonal projection $\tilde{P} : L^2_{per}(\Omega, g) \mapsto \tilde{H}$ and we can get $Q = \tilde{H}^{\perp}$ as

$$Q = CL_{L^2(\Omega)}\{\nabla\phi : \phi \in C^1_{per}(\bar{\Omega}, R)\}$$

which do not depend on the function g.

Therefore, for the given $\mathbf{v} \in L^2_{per}(\Omega, g)$, we can find $\mathbf{u} \in \tilde{H}$ and $\nabla p \in Q$ such that $\mathbf{v} = \mathbf{u} + \nabla p$.

But, for our problem, we are interested in the dynamics on the following spaces;

$$\begin{aligned} H_g &= CL_{L^2(\Omega,g)}\{\mathbf{u} \in C^\infty_{per}(\Omega) \ : \ \nabla \cdot g\mathbf{u} = 0, \ \int_\Omega \mathbf{u} \ d\mathbf{x} = \mathbf{0}\} \\ V_g &= \{\mathbf{u} \in H^1_{per}(\Omega, g) \ : \ \nabla \cdot g\mathbf{u} = 0, \ \int_\Omega \mathbf{u} \ d\mathbf{x} = \mathbf{0}\} \end{aligned}$$

where H_g is endowed with the scalar product and the norm in $L^2(\Omega, g)$, and V_g is the spaces with the scalar product and the norm given by

$$< \mathbf{u}, \mathbf{v} >_{V_g} = \int_\Omega (D_i\mathbf{u} \cdot D_i\mathbf{v}) \ g \ d\mathbf{x} \quad \text{and} \quad \| \mathbf{u} \|^2_{V_g} =< \mathbf{u}, \mathbf{u} >_{V_g},$$

where $\mathbf{x} = (x_1, x_2)$.

Also, for a given $\mathbf{v} \in L^2_{per}(\Omega, g)$, one obtains

$$\mathbf{v} = \mathbf{u} + \frac{\mathbf{k}}{g} + \nabla p, \ \text{for } \mathbf{u} \in H_g, \ \nabla p \in Q, \ \mathbf{k} = \frac{1}{\int_\Omega \frac{1}{g} \ d\mathbf{x}} \int_\Omega \mathbf{v} \ d\mathbf{x}$$

and specially for $g = 1$ one has

$$\mathbf{v} = \mathbf{u} + \mathbf{k} + \nabla p, \ \text{for } \mathbf{u} \in H_1, \ \nabla p \in Q, \ \mathbf{k} = \int_\Omega \mathbf{v} \ d\mathbf{x}.$$

As a result, we can define the orthogonal projection $P_g : L^2_{per}(\Omega, g) \mapsto H_g$, which is similar to the Lerary projection, as $P_g\mathbf{v} = \mathbf{u}$.

Now, throughout this paper we define the g-Laplacian Δ_g by:

$$-\Delta_g\mathbf{u} = -\frac{1}{g}(\nabla \cdot g\nabla)\mathbf{u} = -\Delta\mathbf{u} - \frac{1}{g}(\nabla g \cdot \nabla)\mathbf{u},$$

which is a perturbation of $-\Delta\mathbf{u}$. Then, for $\nu = 1$, (1.1) can be written as

$$\frac{\partial \mathbf{u}}{\partial t} - \Delta_g\mathbf{u} + \frac{1}{g}(\nabla g \cdot \nabla)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \mathbf{f}, \qquad \text{in} \qquad \Omega. \tag{3.1}$$

Thus, by taking the orthogonal projection P_g into (3.1), one obtains

$$\frac{d\mathbf{u}}{dt} + A_g\mathbf{u} + B_g(\mathbf{u}, \mathbf{u}) = \mathbf{q} \qquad \text{on} \qquad H_g, \tag{3.2}$$

where $A_g\mathbf{u} = P_g(-\Delta_g\mathbf{u})$, $B_g(\mathbf{u}, \mathbf{u}) = P_g(\mathbf{u} \cdot \nabla)\mathbf{u}$, $\mathbf{q} = P_g[\mathbf{f} - \frac{1}{g}(\nabla g \cdot \nabla)\mathbf{u}]$. In this paper, we will call the linear operator $A_g\mathbf{u} = P_g(-\Delta_g\mathbf{u})$ as the g-Stokes operator. Also, we obtain the following lemma for the g-Stokes operator, see Roh[7].

Lemma 3.1. *For the g-Stokes operator A_g, the followings hold;*

(1) *The g-Stokes operator A_g is a positive, self adjoint operator with compact inverse, where the domain of A_g, $\mathcal{D}(A_g) = V_g \cap H^2(\Omega, g)$.*

(2) *There exist countable eigenvalues of* A_g *satisfying*

$$0 < \lambda(g) \le \lambda_1 \le \lambda_2 \le \lambda_3 \le ...$$

where $\lambda(g) = \frac{4\pi^2 m}{M}$ *and* λ_1 *is the smallest eigenvalue of* A_g*. In addition, there exist the corresponding collection of eigenfunctions* $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, ...\}$ *forms an orthonormal basis for* H_g*.*

Next, we denote the bilinear operator $B_g(\mathbf{u}, \mathbf{v}) = P_g(\mathbf{u} \cdot \nabla)\mathbf{v}$ and the trilinear form

$$b_g(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \sum_{i,j=1}^{2} \int_\Omega \mathbf{u}_i (D_i \mathbf{v}_j) \mathbf{w}_j g dx,$$

where $\mathbf{u}, \mathbf{v}, \mathbf{w}$ lie in appropriate subspaces of $L^2_{per}(\Omega, g)$ and $D_i = \frac{\partial}{\partial x_i}$.

Then, one obtains

$$\begin{aligned} b_g(\mathbf{u}, \mathbf{v}, \mathbf{w}) &= \sum_{i,j=1}^{2} \int_\Omega \mathbf{u}_i (D_i \mathbf{v}_j) \mathbf{w}_j g dx = \sum_{i,j=1}^{2} \int_\Omega g \mathbf{u}_i (D_i \mathbf{v}_j) \mathbf{w}_j dx \\ &= -\sum_{i,j=1}^{2} \int_\Omega D_i (g \mathbf{u}_i) \mathbf{v}_j \mathbf{w}_j dx - \sum_{i,j=1}^{2} \int_\Omega g \mathbf{u}_i \mathbf{v}_j (D_i \mathbf{w}_j) dx \\ &= -\sum_{i,j=1}^{2} \int_\Omega g \mathbf{u}_i \mathbf{v}_j (D_i \mathbf{w}_j) dx = -b_g(\mathbf{u}, \mathbf{w}, \mathbf{v}), \end{aligned}$$

for sufficient smooth functions $\mathbf{u}, \mathbf{v}, \mathbf{w} \in H_g$ and hence $b_g(\mathbf{u}, \mathbf{v}, \mathbf{w}) = -b_g(\mathbf{u}, \mathbf{w}, \mathbf{v})$ which implies $b_g(\mathbf{u}, \mathbf{v}, \mathbf{v}) = 0$.

Now, we are in the position to see the existence of the solutions of the g-Navier-Stokes equations. See Roh[7] for the proof.

Theorem 3.2. *Let* $\mathbf{f} \in L^\infty(0, \infty; L^2(\Omega, g))$ *be given. Then for every* $\mathbf{u}_0 \in H_g$ *there is precisely one weak solution (of class LH)* $\mathbf{u} = \mathbf{u}(t)$ *on* $[0, \infty)$ *of (3.2), satisfying* $\mathbf{u}(0) = \mathbf{u}_0$*. Moreover one has* $\mathbf{u}(t) \in C[0, \infty; H_g)$*. Also, let* $\mathbf{u} = \mathbf{u}(t)$ *be any weak solution of (3.2) on* $[0, \infty)$ *with initial condition* $\mathbf{u}(0) = \mathbf{u}_0 \in H_g$*. Then for each* $t_0 > 0$*,* $\mathbf{v}(t) = \mathbf{u}(t + t_0)$ *is a strong solution of (3.2) on* $[0, \infty)$ *with initial condition* $\mathbf{v}(0) = \mathbf{u}(t_0)$ *and* $D_t \mathbf{u} \in L^2_{loc}(0, \infty; H_g)$*.*

Now, we assume that the forcing term $\mathbf{f}$ is a time independent function and let $\sigma_w(t, \mathbf{u}_0) = S_w(t)\mathbf{u}_0$ denoted the semiflows on H_g generated by a weak solution on with the data $(\mathbf{u}_0, \mathbf{f})$ where $\mathbf{u}_0 \in H_g$ and $\mathbf{f} \in L^2(\Omega, g)$. Likewise, let $\sigma_s(t, \mathbf{u}_0) = S_s(t)\mathbf{u}_0$ denote the semiflows on V_g generated by a strong solution with the data $(\mathbf{u}_0, \mathbf{f})$, where $\mathbf{u}_0 \in V_g$ and $\mathbf{f} \in L^2(\Omega, g)$.

Theorem 3.3. *Let* $\mathbf{f} \in L^2(\Omega, g)$ *and we assume that* $\| \nabla g \|^2_\infty < \frac{m^3 \pi^2}{M}$*. Then, for* $\mathbf{u}_0 \in H_g$*,* $\sigma_w(t, \mathbf{u}_0) = S_w(t)\mathbf{u}_0$ *is a semiflow on* H_g *which is point dissipative and compact for* $t > 0$*. Also, there exists a global attractor* $\mathcal{A}_w$ *for* $S_w(t)$ *and the semiflow* $S_w(t)$ *is robust at* $\mathcal{A}_w$ *for every* $\mathbf{f} \in L^2(\Omega, g)$*.*

Likewise for $\mathbf{u}_0 \in V_g$*,* $\sigma_s(t, \mathbf{u}_0) = S_s(t)\mathbf{u}_0$ *is a semiflow on* V_g *which is point dissipative and compact for* $t > 0$*. Furthermore, there exists a global attractor* $\mathcal{A}_s$

for $S_s(t)$ and the semiflow $S_s(t)$ is robust at $\mathcal{A}_s$ for every $\mathbf{f} \in L^2(\Omega, g)$. In addition, we note that $\mathcal{A}_s = \mathcal{A}_w$, for fixed $\mathbf{f} \in L^2(\Omega, g)$.

Proof. See Roh[7]. □

Next, we define new set Λ as the following.

Definition 3.4. *Let us define the set Λ with the metric inherited from $W^{1,\infty}(\Omega)$ as $g \in \Lambda$ if*

(1) *$g(\mathbf{x}) \in C^{\infty}_{per}(\Omega)$ and $\int_\Omega \frac{1}{g}\, d\mathbf{x} = 1$ with $0 < m \leq g(x, y) \leq M$, for all $(x, y) \in \Omega$.*
(2) *$\| g \|^2_{W^{1,\infty}} < \frac{m^3 \pi^2}{M}$ and $\| g \|_{W^{2,\infty}} \leq M_0$ for some constant M_0.*

Note that in definition 3.4, the constant function $g = 1$ belong to the set Λ and the condition $\int_\Omega \frac{1}{g}\, d\mathbf{x} = 1$ is to simplify the calculations.

We define $\tilde{\sigma}_w(g, \mathbf{v}, t)$ on H_1 by

$$\tilde{\sigma}_w(g, \mathbf{v}, t) = P_1 \sigma_w(g,\ P_g \mathbf{v},\ t),$$

where $\sigma_w(g, P_g\mathbf{v}, t)$ is a semiflow on the space H_g generated by weak solutions of the equation (3.2) with the initial condition $P_g\mathbf{u}$. Then, $\tilde{\sigma}_w(g, \mathbf{v}, t)$ is a semiflow on H_1 and we have the following robustness theorem. One should note that H_1 and V_1 mean H_g and V_g for $g = 1$.

Theorem 3.5. *Let $\mathbf{f} \in L^2(\Omega)$ and $g \in \Lambda \subset W^{2,\infty}(\Omega)$, where Λ is given in definition 3.4. Then, for every $g \in \Lambda$, $\tilde{\sigma}_w(g, \mathbf{v}, t)$ has a global attractor and the family of the semiflows with respect to g, $\tilde{\sigma}_w(g, \mathbf{v}, t)$, is robust at the global attractor of the semiflow $\tilde{\sigma}_w(1, \mathbf{v}, t)$.*

Proof. See Roh[8]. □

Also, we can define the semiflow on V_1 by

$$\tilde{\sigma}_s(g, \mathbf{v}, t) = P_1 \sigma_s(g,\ P_g \mathbf{v},\ t),$$

where $\sigma_s(g, P_g\mathbf{v}, t)$ is a semiflow on the space V_g generated by the strong solutions of equation (3.2) with the initial condition $P_g\mathbf{v}$. And we get the following theorem, due to Robustness theorem.

Theorem 3.6. *Let $\mathbf{f} \in L^2(\Omega)$ and $g \in \Lambda \subset W^{2,\infty}(\Omega)$, where Λ is given in definition 3.4. Then, for every $g \in \Lambda$, $\tilde{\sigma}_s(g, \mathbf{v}, t)$ has a global attractor and the family of the semiflows with respect to g, $\tilde{\sigma}_s(g, \mathbf{v}, t)$, is robust at the global attractor of the semiflow $\tilde{\sigma}_s(1, \mathbf{v}, t)$.*

Proof. See Roh[8]. □

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