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BANACH SPACE-LIKE PROPERTIES IN C^*-ALGEBRAS INVARIANT UNDER MORITA EQUIVALENCE

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Abstract. We discuss that some Banach space-like properties in C^*-algebras are preserved under Morita equivalence and we show that those properties are characterized by imprimitivity bimodules.

1. Introduction

The notion of Morita equivalence has become a standard tool in recent developments of the theory of C^*-algebras, especially, in K-theory, group C^*-algebras and C^*-crossed products, and actually Morita equivalence gives an equivalence relation between C^*-algebras. It hence would be interesting to investigate what properties in C^*-algebras are preserved under Morita equivalence. In fact, there are already some results along such a line. For example, nuclearity, type I-ness and simplicity of C^*-algebras are preserved under Morita equivalence (see [2, 3.2] for nuclearity; [2, 2.2] for type I-ness; and [19, Theorem 3.22] for simplicity). In this paper, we discuss some Banach space-like properties in C^*-algebras preserved under Morita equivalence.

Let C^*-algebras A and B be Morita equivalent. Then there is a Banach space X called an A-B imprimitivity bimodule associated with both A and B. When some property in A and B is preserved under Morita equivalence, it is natural to expect that X must contain sufficient information on that property of A and B. In such a viewpoint, we characterize some properties in C^*-algebras invariant under Morita equivalence, using imprimitivity bimodules. More precisely, as such properties we deal with the Banach-Saks property and the weak Banach-Saks property in §2, the Dunford-Pettis property in §3 for C^*-algebras respectively, the Radon-Nikodým property for the conjugate spaces of imprimitivity bimodules in §4. In §5, we discuss when every closed A-B-submodule of X is orthogonally complemented in X.

2. The Banach-Saks property and the weak Banach-Saks property

Let A be a C^*-algebra. Recall the definition of a Hilbert C^*-module. By a *left Hilbert A-module*, we mean a left A-module X equipped with an A-valued pairing $\langle \cdot\ ,\ \cdot \rangle$, called an A-valued inner product, satisfying the following conditions:

(a) $\langle \cdot\ ,\ \cdot \rangle$ is sesquilinear. (We make the convention that $\langle \cdot\ ,\ \cdot \rangle$ is linear in the first variable and is conjugate-linear in the second variable.)

(b) $\langle x\ ,\ y \rangle = \langle y\ ,\ x \rangle^*$ for all $x, y \in X$.

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(c) $\langle ax\ ,\ y\rangle = a\langle x\ ,\ y\rangle$ for all $x, y \in X$ and $a \in A$.

(d) $\langle x\ ,\ x\rangle \geqq 0$ for all $x \in X$, and $\langle x\ ,\ x\rangle = 0$ implies that $x = 0$.

(e) X is complete with respect to the norm $\|x\| = \|\langle x\ ,\ x\rangle\|^{\frac{1}{2}}$.

If X satisfies the following additional condition:

(f) the closed linear span of $\{\langle x\ ,\ y\rangle \mid x, y \in X\}$ coincides with A,

then X is said to be *full.*

Let B be a C^*-algebra. Right Hilbert B-modules are defined similarly except that we require that B should act on the right of X, that $\langle\cdot\ ,\ \cdot\rangle$ is conjugate-linear in the first variable, and that $\langle x\ ,\ yb\rangle = \langle x\ ,\ y\rangle b$ for all $x, y \in X$ and $b \in B$.

Let A and B be C^*-algebras. We denote by ${}_A\langle\cdot\ ,\ \cdot\rangle$ the A-valued inner product on the left Hilbert A-module and by $\langle\cdot\ ,\ \cdot\rangle_B$ the B-valued inner product on the right Hilbert B-module, respectively. By an *A-B imprimitivity bimodule* X, we mean a full left Hilbert A-module and full right Hilbert B-module X satisfying

(g) ${}_A\langle xb\ ,\ y\rangle = {}_A\langle x\ ,\ yb^*\rangle$ and $\langle ax,\ y\rangle_B = \langle x\ ,\ a^*y\rangle_B$ for all $x, y \in X, a \in A$ and $b \in B$;

(h) ${}_A\langle x\ ,\ y\rangle{\cdot}z = x{\cdot}\langle y\ ,\ z\rangle_B$ for all $x, y, z \in X$.

Two C^*-algebras A and B are said to be *Morita equivalent* if there exists an A-B-imprimitivity bimodule. We remark that in this paper, Morita equivalence means strong Morita equivalence in the sense of Rieffel. The reader is referred to [19] for Hilbert C^*-modules and Morita equivalence.

Now we briefly review the Banach-Saks property in Banach spaces. In [1], Banach and Saks showed that every bounded sequence in $L^p([0,1])$ with $1 < p < \infty$ has a subsequence whose arithmetic means converge in the norm topology. More generally, if every bounded sequence $\{x_n\}$ in a Banach space X has a subsequence $\{x_{n(k)}\}$ such that

$$\lim_{k\to\infty} \|\frac{1}{k}(x_{n(1)} + \cdots + x_{n(k)}) - y\| = 0$$

with some $y \in X$, we say that X has *the Banach-Saks property.* It is known that Banach spaces with the Banach-Saks property are reflexive. It hence follows that $L^1([0,1])$ can not have the Banach-Saks property. Furhtermore it is shown that a C^*-algebra has the Banach-Saks property if and only if it is finite-dimensional (cf. [16]). Using this fact, we establish the following.

Theorem 2.1. *Let two unital C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Then the following conditions are equivalent.*
(1) *A has the Banach-Saks property.*
(2) *B has the Banach-Saks property.*
(3) *X has the Banach-Saks property.*

See [16] for the proof. Applying the above theorem to the linking algebra for an A-B-imprimitivity bimodule X, we obtain the following.

Corollary 2.2. *Let two unital C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Then X has the Banach-Saks property if and only if X is finite-dimensional.*

In the above theorem and corollary, the assumption that both C^*-algebras A and B be unital is essential. In fact, let H be an infinite-dimensional Hilbert space and let $\mathcal{C}(H)$ be the C^*-algebra of all compact operators on H, which is not unital.

Then $\mathbb{C}\cdot 1$ and $\mathcal{C}(H)$ are Morita equivalent through the imprimitivity bimodule H. But $\mathcal{C}(H)$ does not have the Banach-Saks property.

Let X be a Banach space. If given any weakly null sequence $\{x_n\}$ in X, one can extract a subsequence $\{x_{n(k)}\}$ such that

$$\lim_{k\to\infty}\frac{1}{k}\|x_{n(1)}+\cdots+x_{n(k)}\|=0,$$

we say that X has *the weak Banach-Saks property.* It was shown by Szlenk [20] that $L^1([0,1])$ has the weak Banach-Saks property. Furthermore, there is a slightly stronger version of the weak Banach-Saks property introduced by Nuñez [18]. We say that a Banach space X has *the uniform weak Banach-Saks property* if there is a null sequence $\{\delta_n\}$ of positive real numbers such that, for any weakly null sequence $\{x_n\}$ in X with $\|x_n\|\leqq 1$ and for any natural number k, there exist natural numbers $n(1)<n(2)<\cdots<n(k)$ such that

$$\frac{1}{k}\|x_{n(1)}+\cdots+x_{n(k)}\|<\delta_k.$$

Let X be a Banach space and let $c_0(X)$ be the Banach space of all null sequences in X. It is then shown that X has the uniform weak Banach-Saks property if and only if $c_0(X)$ has the weak Banach-Saks property ([18, Theorem 3]).

For a C^*-algebra A, we denote by $\widehat{A}$ the spectrum of A, that is, the set of (unitary) equivalence classes $[\pi]$ of nonzero irreducible representations π of A equipped with the Jacobson topology. The reader is referred to [9] for the spectrum of a C^*-algebra.

Recently, Chu [5] has studied C^*-algebras with the weak Banach-Saks property in detail, as a noncommutative extension of characterisations of the Banach space, of complex continuous functions on a compact Hausdorff space, with the weak Banach-Saks property. Actually he has obtained the following characterization of C^*-algebras with the weak Banach-Saks property.

Theorem ([5, Theorem 2]). *Let A be a C^*-algebra. Then the following conditions are equivalent:*
(1) *A has the weak Banach-Saks property.*
(2) *A has the uniform weak Banach-Saks property.*
(3) *A is scattered and $c_0(A)$ does not contain an isometric copy of $C_0(\omega^\omega)$, where ω^ω denotes the set $[0,\ \omega^\omega)$ of ordinals preceding ω^ω with the order topology.*
(4) *A is scattered and does not contain an isometric copy of $C_0(\omega^\omega)$.*
(5) *A is of type I and $\widehat{A}^{(k)}$ is empty for some natural number k, where $\widehat{A}^{(0)}=\widehat{A}$, the spectrum of A, and $\widehat{A}^{(n)}$ is the n-th derived set of $\widehat{A}$, consisiting of the accumulation points of $\widehat{A}^{(n-1)}$.*
(6) *There exists some natural number k such that $\sigma(a)^{(k)}$ is empty for every self-adjoint $a\in A$, where $\sigma(a)$ denotes the spectrum of a.*

The following theorem can be proved with [11, Thoerem 2.3] and [15, Theorem 2.2] combined.

Theorem 2.3. *Let two C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Consider the following conditions :*
(1) *A has the weak Banach-Saks property.*
(2) *B has the weak Banach-Saks property.*
(3) *A has the uniform weak Banach-Saks property.*
(4) *B has the uniform weak Banach-Saks property.*
(5) *X has the uniform weak Banach-Saks property.*
(6) *X has the weak Banach-Saks property.*
Then we have the implications $(1) \Longleftrightarrow (2) \Longleftrightarrow (3) \Longleftrightarrow (4) \Longrightarrow (5) \Longrightarrow (6)$. *If either A or B is unital, then conditions* $(1) - (6)$ *are equivalent.*

3. The Dunford-Pettis property

Let X be a Banach space and denote by X^* the dual space of X. We say that the Banach space X has the *Dunford-Pettis property* if every weakly compact operator on X is completely continuous, that is, such an operator takes weakly Cauchy sequences into norm Cauchy sequences. Equivalently, X has the Dunford-Pettis property if, for any weakly null sequence $\{x_n\} \subset X$ and any weakly null sequence $\{\varphi_n\} \subset X^*$, we have $\varphi_n(x_n) \to 0$ $(n \to \infty)$.

The above definition was due to Grothendieck and was originated from a classical result of Dunford-Pettis which says that all L^1-spaces have this property. Note that a typical example of Banach spaces with the Dunford-Pettis property is a Banach space $C(K)$ of all continuous functions on a compact Hausdorff space K. The reader is referred to [7] for more details of the Dunford-Pettis property.

Since every C^*-algebra is a Banach space, it is natural to consider C^*-algebras with the Dunford-Pettis property and to characterize them. In fact, some characterizations of C^*-algebras with the Dunford-Pettis property were done by Chu [4], Chu-Iochum [6], Hamana [10], and so on. In particular, a crucial characterization is that a C^*-algebra A has the Dunford-Pettis property if and only if every irreducible representation of A is finite-dimensional (see [4]). We will employ this result to obtain the following theorem (see [14, Theorem 3.2] for the proof).

Theorem 3.1. *Let unital C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Then the following conditions* $(1) - (3)$ *are equivalent.*
(1) *A has the Dunford-Pettis property.*
(2) *B has the Dunford-Pettis property.*
(3) *X has the Dunford-Pettis property.*

In the above theorem, the assumption that both C^*-algebras A and B be unital is essential. If H is an infinite-dimensional Hilbert space, then the C^*-algebra $\mathcal{C}(H)$ of all compact operators on H does not have the Dunford-Pettis property. But $\mathbb{C}\cdot 1$ and $\mathcal{C}(H)$ are Morita equivalent.

We end this section by stating a typical example of C^*-algebras with the Dunford-Pettis property. Let $k = 1, 2, \cdots, m$. Let Ω_k be a compact Hausdorff space and M_{n_k} be the matrix algebra of $n_k \times n_k$ complex matrices. Consider the C^*-algebra $\oplus_{k=1}^m (C(\Omega_k) \otimes M_{n_k})$ of the finite direct sum of the C^*-tensor products $\{C(\Omega_k) \otimes M_{n_k}\}_k$. Then this C^*-algebra has the Dunford-Pettis property.

4. The Radon-Nikodým property in Hilbert C^*-modules

We first recall that a C^*-algebra A is said to be a *scattered* C^*-algebra if every positive linear functional on A is the countable sum of pure positive linear functionals on A, equivalently, A is of type I and the spectrum $\widehat{A}$ of A is a scattered topological space. We say that a C^*-algebra is *σ-unital* if it contains a strictly positive element. It is well-known that a C^*-algebra is σ-unital if and only if it has a countable approximate identity. Thus every separable C^*-algebra is always σ-unital. But note that σ-unital C^*-algebras are not necessarily separable.

Our starting point in this section is the crucial result, which was shown by Chu [3], that *A is a scattered C^*-algebra if and only if A^* has the the Radon-Nikodým property.* In fact, we shall show that if A or B is a scattered C^*-algebra, then X^* has the Radon-Nikodým property, and conversely that if A or B is σ-unital and if X^* has the Radon-Nikodým property, then A and B are scattered C^*-algebras. The difficult part of this result is to prove that the Radon-Nikodým property in X^* implies scatteredness of A and B.

Here we recall the definition of the Radon-Nikodým property. We say that a Banach space X has the *Radon-Nikodým property* if for any finite measure space (Ω, Σ, μ) and any μ-continuous vector measure $L : \Sigma \to X$ of bounded total variation, there exists a Bochner integrable function $f : \Omega \to X$ such that

$$L(E) = \int_E f(\omega) d\mu(\omega)$$

for all E in Σ. The reader is referred to [8] for the details of the Radon-Nikodým property. Let X be a Banach space. Then the crucial result obtained by Stegall and Uhl is that *the dual space X^* has the Radon-Nikodým property if and only if every separable subspace of X has a separable dual* (see [8], p.28 and the references cited therein) . In fact, we will use such a result to prove our theorem below.

Now we are in a position to establish the main result (see [12] for the proof).

Theorem 4.1. *Let two C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Consider the following conditions.*
(1) *A is a scattered C^*-algebra.*
(2) *B is a scattered C^*-algebra.*
(3) *A^* has the Radon-Nikodým property.*
(4) *B^* has the Radon-Nikodým property.*
(5) *X^* has the Radon-Nikodým property.*
Then we have the implications (1) $\Longleftrightarrow$ (2) $\Longleftrightarrow$ (3) $\Longleftrightarrow$ (4) $\Longrightarrow$ (5). *If either A or B is σ-unital, then conditions* (1)-(5) *are equivalent.*

Let A be a C^*-algebra. As mentioned before, A is a scattered C^*-algebra if and only if A^* has the the Radon-Nikodým property ([3]). In the case where A is σ-unital, the above theorem generalizes this result. We consider the case where $A = B$ in the above theorem. Then we can take A itself as an A-A-imprimitivity bimodule X. In fact, A can be regarded as an A-A-imprimitivity bimodule, for the bimodule structure given by the multiplication in A, with ${}_A\langle a\ ,\ b\rangle = ab^*$ and $\langle a\ ,\ b\rangle_A = a^*b$ for $a, b \in A$ (see [19, Example 3.5]).

5. Complemented submodules in Hilbert C^*-modules

Although a Hilbert C^*-module is a generalization of a Hilbert space, we cannot, a prioli, expect that Hilbert C^*-modules behave like Hilbert spaces, and in some ways they do. In fact, as is mentioned below, there is an essential way in which Hilbert C^*-modules differ from Hilbert spaces. Let X be a Hilbert C^*-module equipped with inner product $\langle \cdot \, , \, \cdot \rangle$ taking values in the C^*-algebra and let Y be a closed subspace Y of X. We denote by $Y^{\perp}$ the orthogonally complemented subspace of Y in X, i.e.,

$$Y^{\perp} = \{ \, x \in X \mid \langle x \, , \, y \rangle = 0 \text{ for all } y \in Y \, \}.$$

Then $Y^{\perp}$ is also a closed subspace of X, and $(Y^{\perp})^{\perp}$ is usually larger than Y. In general, X is not equal to the direct sum $Y \oplus Y^{\perp}$ of Y and $Y^{\perp}$. We say that a closed subspace Y of a Hilbert A-module X is *complemented* in X if X coincides with $Y \oplus Y^{\perp}$. Throughout this section, by a complemented subspace of X we mean one that is *orthogonally* complemented. Since the whole theory of Hilbert spaces and their linear operators is based on the use of the orthogonally complemented subspaces, it is clear that there will be obstacles in developing a theory of Hilbert C^*-modules analogous to that of Hilbert spaces. Hence, when it is necessary to obtain an analogous theory which works for Hilbert C^*-modules, what we must first know is when *every* closed submodule of a Hilbert C^*-module is complemented. At present, as for related results, there are only a few results on what kind of closed submodule is complemented in a Hilbert C^*-module (see, for example, [17, Chapter 3]). As far as we know, however, there are no answers to the problem of when every closed submodule of a Hilbert C^*-module is complemented. The aim of this section is to provide a few of sufficiently satisfactory answers to such a problem.

Now we give examples that $Y \oplus Y^{\perp} \neq X$. Put $A = C([0,1])$ which is the C^*-algebra of all continuous functions on the interval [0, 1]. Take $X = C([0,1])$ as a Hilbert A-module and $Y = \{f \in X \mid f(0) = 0\}$ as a closed A-submodule of X. Then it is easily verified that $Y^{\perp} = \{0\}$, which implies that $Y \oplus Y^{\perp} \neq X$.

As another exmple, take $A = C([0,1])$ above and put $I = \{f \in A \mid f(0) = 0\}$. Consider $X = A \oplus I$ as a Hilbert A-module and $Y = \{(f,f) \mid f \in I\}$ as a closed A-submodule Y of X. Then $Y^{\perp} = \{(g, -g) | g \in I\}$. Hence $Y \oplus Y^{\perp} = I \oplus I \neq X$.

The following theorem is one of answers to the problem mentioned above. See [13] for the proof.

Theorem 5.1. *Let two C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Consider the following conditions:*
(1) *The spectrum $\widehat{A}$ of A is discrete in the Jacobson topology.*
(2) *The spectrum $\widehat{B}$ of B is discrete in the Jacobson topology.*
(3) *Every closed A-B-submodule of X is comlemented in X.*
Then we have $(1) \Longleftrightarrow (2) \Longrightarrow (3)$. *If either $\widehat{A}$ or $\widehat{B}$ is a T_1-space, then conditions* $(1) - (3)$ *are equivalent.*

Let A be a C^*-algebra and let X be a Hilbert A-module with A-valued inner product $\langle \cdot \, , \, \cdot \rangle$. For convenience, without loss of generality we suppose that X is a right Hilbert A-module. We define the linear operator $\theta_{x,y}$ by $\theta_{x,y}(z) = x \cdot \langle y \, , \, z \rangle$ for all $x, y, z \in X$. We denote by $\mathcal{K}(X)$ the C^*-algebra generated by the set $\{\theta_{x,y} \mid x \, y \in X \, \}$ (see [19, Proposition 2.21 and Lemma 2.25]). Then X is a $\mathcal{K}(X)$-A-Hilbert bimodule. If X is a full left Hilbert A-module, then it is $\mathcal{K}(X)$-A-imprimitivity bimodule ([19, Proposition 3.8]).

Corollary 5.2. *Let A be a C^*-algebra and let X be a Hilbert C^*-module over A. We denote by I the closed ideal of A generated by $\langle X\ ,\ X\rangle$. Consider the following conditions:*
(1) *The spectrum $\widehat{A}$ of A is discrete in the Jacobson topology.*
(2) *The spectrum $\widehat{I}$ of I is discrete in the Jacobson topology.*
(3) *Every closed $\mathcal{K}(X)$-I-submodule of X is comlemented in X.*

Then we have (1) $\Longrightarrow$ (2) $\Longrightarrow$ (3). *If $\widehat{I}$ is a T_1-space, then conditions* (2) *and* (3) *are equivalent.*

In Theorem 5.1 and Corollary 5.2, it is possible to replace the spectra of C^*-algebras by the primitive spectra. In fact, we have the following ([13, Theorem 2.6 and Corollary 2.7]).

Theorem 5.3. *Let two C^*-algebras A and B be Morita equivalent and let X be an A-B-imprimitivity bimodule. Consider the following conditions:*
(1) *The primitive spectrum Prim(A) of A is discrete in the Jacobson topology.*
(2) *The primitive spectrum Prim(B) of B is discrete in the Jacobson topology.*
(3) *Every closed A-B-submodule of X is comlemented in X.*
Then we have (1) $\Longleftrightarrow$ (2) $\Longrightarrow$ (3). *If either Prim(A) or Prim(B) is a T_1-space, then conditions* $(1) - (3)$ *are equivalent.*

Corollary 5.4. *Let A be a C^*-algebra and let X be a Hilbert C^*-module over A. We denote by I the closed ideal of A generated by $\langle X\ ,\ X\rangle$. Consider the following conditions:*
(1) *The primitive spectrum Prim(A) of A is discrete in the Jacobson topology.*
(2) *The primitive spectrum Prim(I) of I is discrete in the Jacobson topology.*
(3) *Every closed $\mathcal{K}(X)$-I-submodule of X is comlemented in X.*

Then we have (1) $\Longrightarrow$ (2) $\Longrightarrow$ (3). *If Prim(I) is a T_1-space, then conditions* (2) *and* (3) *are equivalent.*

Example 5.5. If a C^*-algebra A is finite-dimensional, then it is isomorphic to the direct sum of some matrix algebras. Hence the spectrum $\widehat{A}$ of A is discrete. Let X be a full right A-Hilbert module. Then, by Corollary 5.2, every closed $\mathcal{K}(X)$-A-submodule of X is comlemented in X.

More generally, if A is a dual C^*-algebra, then the spectrum $\widehat{A}$ of A is discrete. Recall here that a C^*-algebra is said to be *dual* if it is isomorphic to a C^*-subalgebra of the C^*-algebra of all compact operators on some Hilbert space (cf. [9, 4.7.20]). Note that a separable C^*-algebra is dual if and only if its spectrum is discrete in the Jacobson topology. Let A be a dual C^*-algebra and let X be a full right A-Hilbert module. Then every closed $\mathcal{K}(X)$-A-submodule of X is comlemented in X.

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